3.7

Note Title 9/19/2007

Derivatives of Log functions

$$\frac{d}{dx}(\log_a x) = \frac{1}{x \ln a}$$

ex. $\frac{d}{dx} \log_5 x = \frac{1}{x \ln 5}$

With Chain Rule

ex $\frac{d}{dx} \log_5 x^2 = \frac{1}{x \ln 5} * 2x$

$$= \frac{2x}{x \ln 5}$$

$$\frac{d}{dx} \ln x = \frac{1}{x}$$

ex $\frac{d}{dx} \ln(\sqrt{x}) = \frac{1}{\sqrt{x}} * \frac{1}{2} x^{-1/2}$

$$= \frac{1}{2x}$$

In general,

$$\frac{d}{dx}(\ln u) = \frac{1}{u}\frac{du}{dx} \quad \text{or} \quad \frac{d}{dx}[\ln g(x)] = \frac{g'(x)}{g(x)}$$

$$\frac{d}{dx}\ln|x| = \frac{1}{x}$$

Logarithmic Differentiation

1. Take Ln of both sides of an equation $y = f(x)$ and use the Laws of Logs to simplify
2. Differentiate implicitly with respect to x
3. Solve for y'

ex. $y = \sqrt{x}\, e^{x^2}(x^2+1)^{10}$

$$\ln y = \ln\left(\sqrt{x}\, e^{x^2}(x^2+1)^{10}\right)$$

$$\ln y = \ln(\sqrt{x}) + \ln e^{x^2} + \ln(x^2+1)^{10}$$

$$\ln y = \tfrac{1}{2}\ln x + x^2 + 10\ln(x^2+1)$$

$$\frac{1}{y}y' = \frac{1}{2x} + 2x + 20x\frac{1}{x^2+1}$$

$$y' = y\left(\frac{1}{2x} + 2x + \frac{20}{x^2+1}\right)$$

$$y' = \sqrt{x}\, e^{x^2}(x^2+1)^{10}\left(\frac{1}{2x} + 2x + \frac{20}{x^2+1}\right)$$