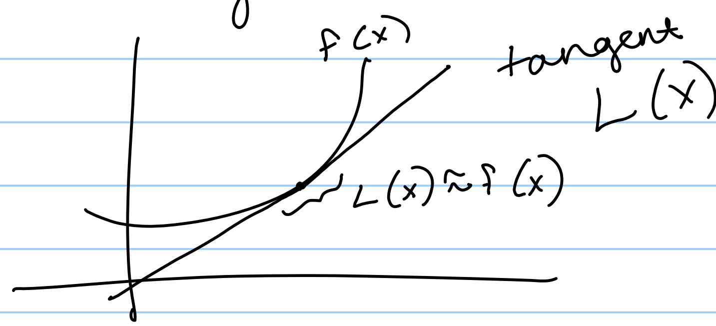
3.8

Note Title 9/24/2007

Linear Approximation

idea: sometimes it is difficult (or impossible) to find $f(x)$ for some number(s) but we know that the tangent line is close to $f(a)$ when x is close to a. So we can use the tangent line to approximate



Need an equation for $L(x)$

→ have point $(a, f(a))$

→ and slope $f'(a)$

$y - f(a) = f'(a)(x-a)$

Use $L(x)$ instead of y and solve

$$L(x) = f(a) + f'(a)(x-a)$$

called <u>linearization of f at a</u>

ex. find Linearization of $f(x) = x^2$ at $a = 1$

$L(x) = f(a) + f'(a)(x-a)$

$L(x) = 1 + 2(x-1)$

$L(x) = 2x - 1$

Now, approximate $f(1.001)$

$f(1.001) \approx L(1.001) = 2(1.001) - 1$

$= 2.002 - 1$

$= 1.002$

So $f(1.001) \approx 1.002$

ex Use linear approximation to estimate $1/1002$

$f(x) = 1/x$ and $a = 1000$

$f'(x) = -x^{-2} = -1/x^2 \Rightarrow f'(1000) = -\frac{1}{1000000}$

$L(x) = f(a) + f'(a)(x-a)$

$L(x) = \frac{1}{1000} - \frac{1}{100000}(x - 1000)$

$= \frac{1}{1000} - \frac{1}{100\,000}x + \frac{1}{1000}$

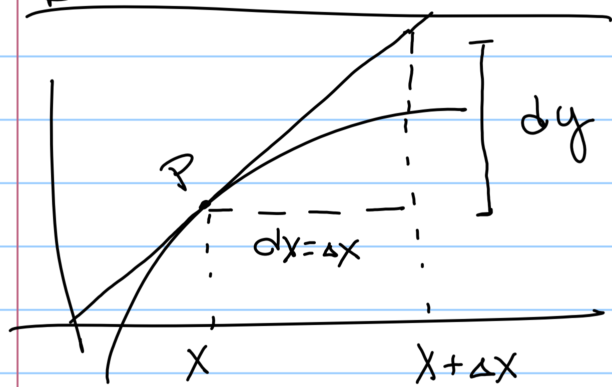
$= -\frac{x}{1000000} + \frac{2}{1000}$

$L(x) = \frac{-x + 2000}{1000000}$

$L(1002) = \frac{-1002 + 2000}{1000000}$

$= \frac{998}{1000000} = .000998$

Differentials



δx and Δy is the change on f

dx and dy is the change in tangent

called <u>differentials</u>

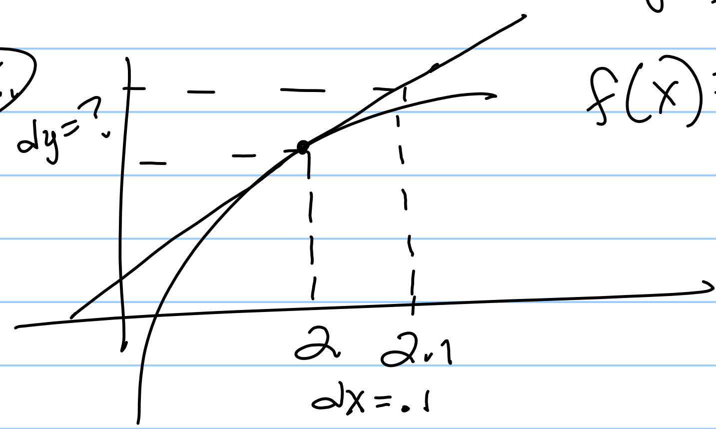
We know $f'(x) = \frac{dy}{dx}$ (since f' is slope of tangent)

We can solve this to get

$$dy = f'(x)\,dx$$

This is helpful in finding error!
(& other things)

ex. $f(x) = -(x-3)^2 + 4$



I measure x and get 2.1 but I should have gotten 2 oops! ☺
So I'm off by 0.1

This makes my y off too but by how much?

$dy = f'(x)\,dx$

$f'(x) = -2(x-3)$

$f'(2) = -2(2-3)$

$= 2$

$dy = 2 * .1$

$dy = .2$

So my y is off by 0.2

relative error - how much is your error in terms of whole answer given by,

$$\frac{dx}{x} \quad dy$$

ex from above

relative error is $\frac{.1}{2} * .2 = .01$

Percentage errors - relative error as a %

ex 1%