Note Title 10/29/2007

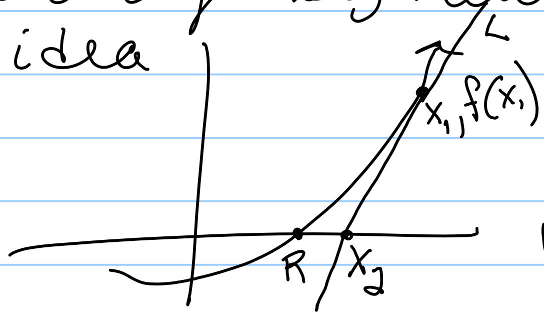
4.8

Newton's Method

Say we want to find the solution to a really messy eqn we could
- graph & zoom to find intercepts
- use solver

But how does our calculator find the answer(s)?
Usually by Newton's Method!

idea



Say, we are trying to find root, R.
We "guess" the answer is x_1. Then draw tangent line thru x_1 (labeled L)
find x-intercept of the tangent line (labeled x_2)
which is easy to find since L is a line.

remember linearization

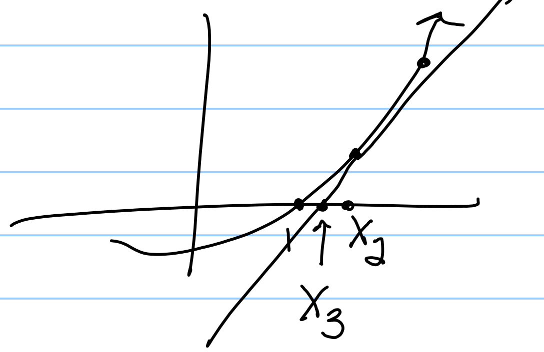
an eqn for L is $y - f(x_1) = f'(x_1)(x - x_1)$

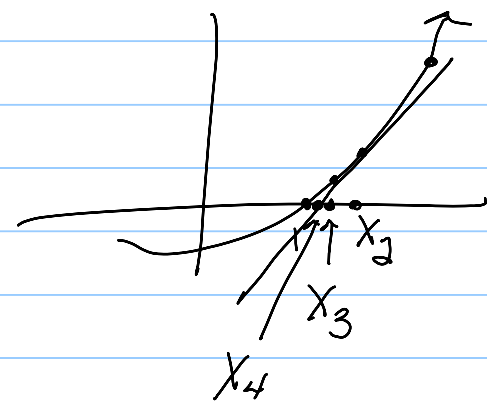
since $(x_2, 0)$ is on L $\quad 0 - f(x_1) = f'(x_1)(x_2 - x_1)$

$$\Rightarrow x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} \qquad \text{(assuming } f'(x_1) \neq 0\text{)}$$

Now use x_2 as our "guess"

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} \qquad x_4 = x_3 - \frac{f(x_3)}{f'(x_3)} \quad \ldots$$





. . .

In general,

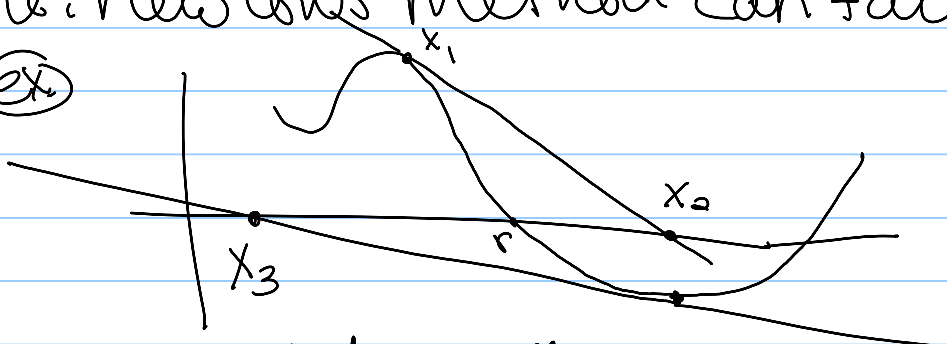
$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

if $x_n \to r$ as $n \to \infty$ we say the sequence converges & write

$$\lim_{n\to\infty} x_n = r$$

Note: Newton's Method can fail

ex.



we needed a better x_1

ex. find x_3 using $x_1 = -1$ for $x^5 + 2 = 0$

$f(x) = x^5 + 2$

$f'(x) = 5x^4$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$x_2 = -1 - \frac{1}{5}$$

$$= -6/5$$

$$x_3 = -6/5 - \frac{f(-6/5)}{f'(-6/5)} \approx -1.1529$$

ex. #14 pg 326 find positive root (to 3 decimal places) of

$$x^2(4 - x^2) = \frac{4}{x^2+1}$$

$$4x^2 - x^4 - \frac{4}{x^2+1} = 0$$

$f(x) = 4x^2 - x^4 - \frac{4}{x^2+1}$

$f'(x) = 8x - 4x^3 + \frac{8x}{(x^2+1)^2}$

guess $x_1 = .8$

$$x_2 \approx .8 - \frac{-.2886}{6.7315} \approx .842873$$

$$x_3 \approx .842873 - \frac{-.0016}{6.6526} \approx .843114$$

$$x_4 \approx .843114 - \frac{.00004}{6.652} \approx .843108$$