4.2 Maximum and Minimum Values

One of the most useful applications in differential calculus are *optimization problems* in which we are called upon to find the best way of doing something. For example, a company which makes canned food may want to know the shape of a can that will minimize their manufacturing costs. In order to solve problems like these we must be able to find the maximum or minimum values of a function.

We begin with some definitions…

Def

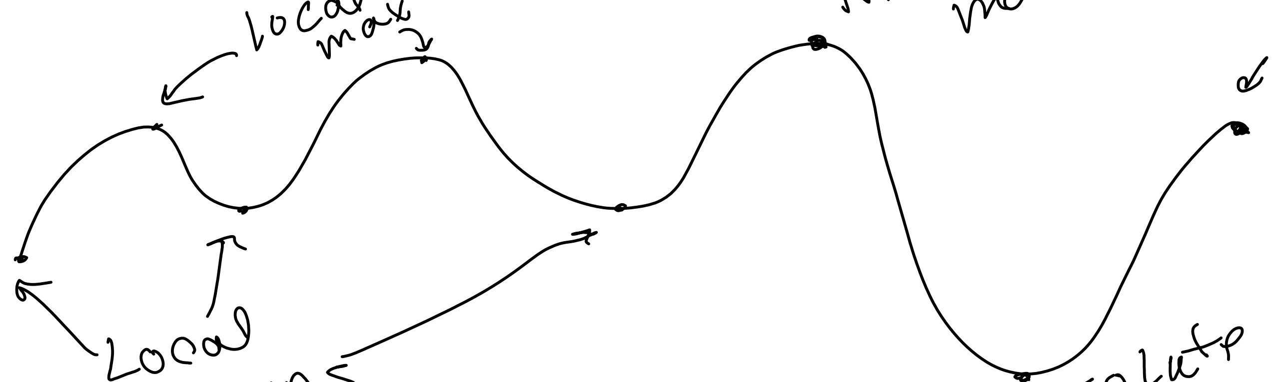
A function f has an **ABSOLUTE MAXIMUM** at c if $f(c) \geq f(x)$ for all x in the domain D.

We call $f(c)$ the MAXIMUM VALUE of f.

A function f has an **ABSOLUTE MINIMUM** at c if $f(c) \leq f(x)$ for all x in the domain D.

In this case we call $f(c)$ the MINIMUM VALUE of f.

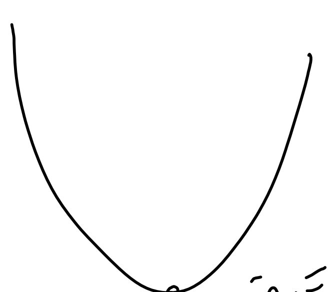
The maximum and minimum values are called **EXTREME VALUES**.

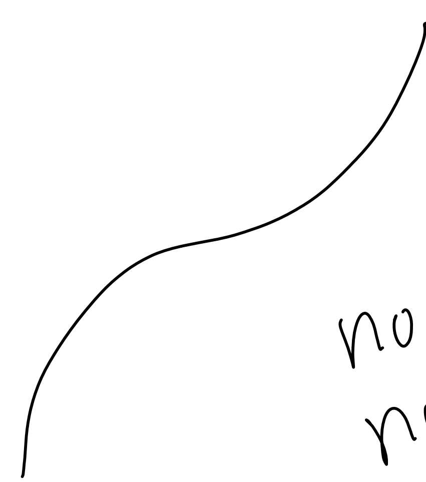


Def

A function f has an **LOCAL MAXIMUM** at c if $f(c) \geq f(x)$ for all x in an open interval **near** c.

A function f has an **LOCAL MINIMUM** at c if $f(c) \leq f(x)$ for all x in an open interval **near** c.

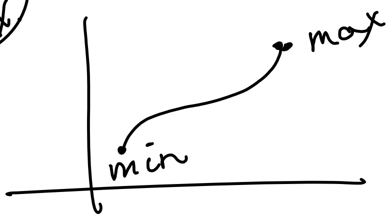




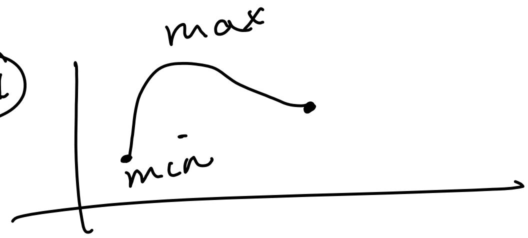
So, it looks like some functions have extreme values and others do not. Is a function ever guaranteed to possess extreme values?

<u>Extreme Value Theorem</u>
If f is continuous on a closed interval $[a, b]$, then f attains an absolute maximum value and an absolute minimum value on $[a, b]$.

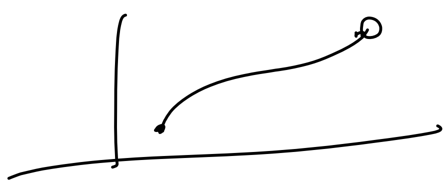
ex



ex



must be closed



Given a function, how do we go about finding extreme values?

<u>Def</u>
A **CRITICAL NUMBER** of a function f is a number c such that $f'(c) = 0$ or $f'(c)$ dne

ex. find critical #s of $f(x) = \sqrt{x^2+2x} = (x^2+2x)^{1/2}$

$f'(x) = \dfrac{2x+2}{2\sqrt{x^2+2x}}$ $\quad f'(x) = 0$ when $2x+2=0 \Rightarrow x=-1$

$f'(x)$ dne when $x^2+2x=0 \Rightarrow x = 0, -2$

critical #s $0, -1, -2$

Here is how we find Absolute Maximum and Minimum Values on a closed interval:

Suppose f is continuous on a closed interval $[a, b]$.

1. Find all critical numbers of f on the interval $[a, b]$, and find the value of f at each of these numbers.

2. Evaluate $f(a)$ and $f(b)$.

3. The largest of the values from steps 1 and 2 is your Absolute Maximum value and the smallest of the values is your Absolute Minimum value.

Ex) #46 p276 Find the absolute maximum and absolute minimum values of f on the given interval.

$$f(x) = x - 2\cos x, \quad [-\pi, \pi]$$

1. $f'(x) = 1 + 2\sin(x)$

$f'(x) = 0$ when $\sin x = -\frac{1}{2}$

$\Rightarrow x = -\frac{\pi}{6}$ and $-\frac{5\pi}{6}$

$x = -.523599$ and -2.61799

$f'(x)$ everywhere defined.

$f\left(-\frac{\pi}{6}\right) = -2.25565$

$f\left(-\frac{5\pi}{6}\right) = -.885943$

2. $f(-\pi) = -1.14159$

$f(\pi) = 5.14159$

3. Abs. Max $= 5.14159$ at $x = \pi$

Abs Min $= -2.25565$ at $x = -\pi/6$