5.3

Note Title 11/28/2007

The integral is the antiderivative

Evaluation Thm

If f is continuous on $[a,b]$ then
$$\int_a^b f(x)\,dx = F(b) - F(a)$$
where F is the antiderivative of f.

ex. In §5.2 we found $\int_0^4 x^2+x-2\,dx$ using Riemann Sums... now

$$\int_0^4 x^2+x-2\,dx = \frac{1}{3}x^3+\frac{1}{2}x^2-2x\Big]_0^4$$
$$= \left[\frac{1}{3}(4)^3+\frac{1}{2}(4)^2-2(4)\right]-\left[\frac{1}{3}(0)^3+\frac{1}{2}(0)^2-2(0)\right]$$
$$= \frac{64}{3}+8-8-0$$
$$= 64/3 = 21.\overline{3}$$

note: this only works when f is continuous on $[a,b]$

Indefinite Integrals

$\int f(x)\,dx = F(x)$ means $F'(x) = f(x)$

ex. $\int \cos x\,dx = \sin x + C$

since $\sin x + C$ is the antiderivative of $\cos x$

"Know" Indefinite Integrals on page 369.

ex. $\int_0^1 \frac{4}{x^2+1}\,dx = 4\int_0^1 \frac{1}{x^2+1}\,dx = 4\tan^{-1}x\Big]_0^1$

$= 4\left[\tan^{-1}(1) - \tan^{-1}(0)\right]$

$= 4\left[\frac{\pi}{4} - 0\right] = \pi$

Applications

Net Change Thm

The integral of a rate of change is the net change:

$$\int_a^b F'(x)\,dx = F(b) - F(a)$$

ex. If $v(t)$ is velocity then

$$\int_a^b v(t)\,dt = s(b) - s(a)$$

which is the net change in position

(recall, §5.1 example about riding your bike)

ex. #50

A honeybee population starts with 100 bees and increases at a rate of $n'(t)$ bees per week. What does $100 + \int_0^{15} n'(t)\,dt$ represent?

The # of bees after 15 weeks, net change in # of bees

See examples on pg 371-372