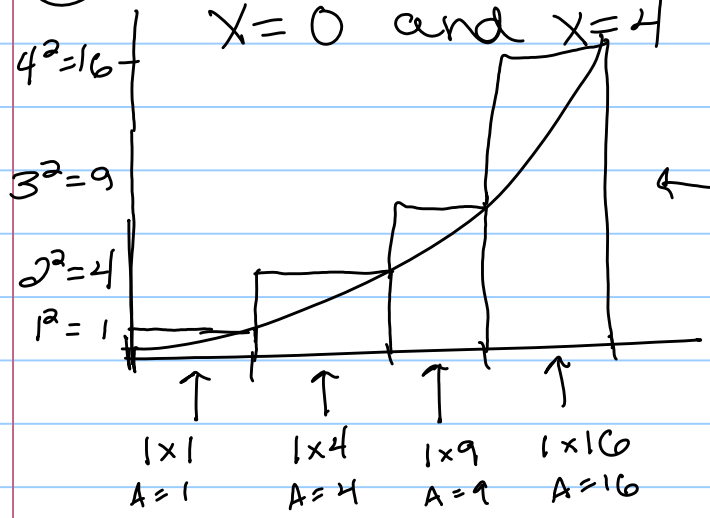
5.1

Note Title 11/26/2007

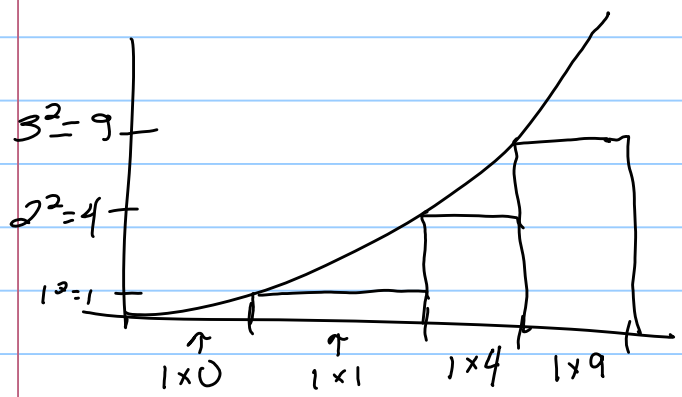
Area

(ex) find area below $y = x^2$ between $x = 0$ and $x = 4$



← called approximating rectangles

$R_4 = 1 + 4 + 9 + 16 = 30$



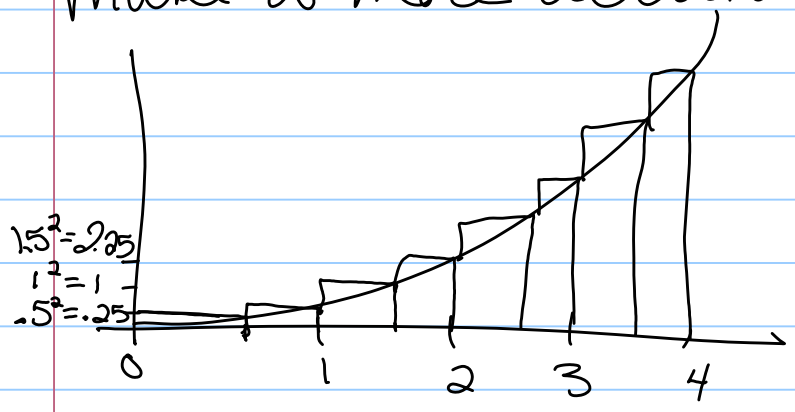
over

under

$L_4 = 0 + 1 + 4 + 9 = 14$

So $14 < A < 30$

Make it more accurate



$$R_8 = .5 * .5^2 + .5 \cdot 1^2 + .5 \cdot 1.5^2 + .5 \cdot 2^2 + .5 \cdot 2.5^2 + \cdots + .5 \cdot 4^2$$
$$= .5(.5^2 + 1^2 + 1.5^2 + 2^2 + 2.5^2 + \cdots + 4^2)$$
$$= .5(51) = 25.5$$

$$L_8 = .5 \cdot 0 + .5 \cdot .5^2 + .5 \cdot 1^2 + .5 \cdot 1.5^2 + \cdots + .5 \cdot 3.5^2$$
$$= .5(35) = 17.5$$

So $17.5 < A < 25.5$

If we found R_{16} and L_{16} be even better or R_{32} and L_{32} or R_{100}, L_{100}, or $\lim_{n\to\infty} R_n$

Defn The area A of a region beneath the graph of f is the limit of the sum of area of approximating rectangles

(f is continuous)

$$A = \lim_{n\to\infty} R_n = \lim_{n\to\infty} \left[f(x_1)\Delta x + f(x_2)\Delta x + \cdots + f(x_n)\Delta x \right]$$

Doesn't need to be right endpoints (could be left or middle or any point)

Sigma notation

$$\sum_{i=1}^{n} f(x_i)\Delta x = f(x_1)\Delta x + f(x_2)\Delta x + \cdots + f(x_n)\Delta x$$

Distance

(ex) You ride your bike for 20 sec at the following speeds How far did you go?

t	0	5	10	15
$V(t)$	5	10	12	12

distance = rate * time

$= 5*5 + 10*5 + 12*5 + 12*5 = 195$ m

[since you went 5 m/s for 5 sec