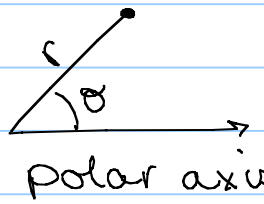
H1

Note Title 2/11/2008

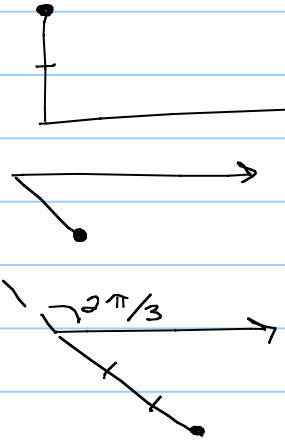


represent points as (r, θ)

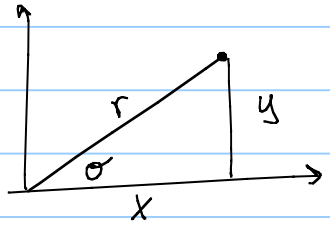
(ex.) Plot $(2, \pi/2)$

Plot $(1, -\pi/3)$

Plot $(-3, 2\pi/3)$



Converting Polar to Cartesian (or vice-versa)



$\cos\theta = \frac{x}{r} \Rightarrow x = r\cos\theta$

$\sin\theta = \frac{y}{r} \Rightarrow y = r\sin\theta$

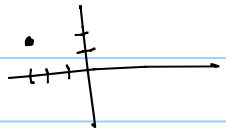
$r^2 = x^2 + y^2 \quad \tan\theta = \frac{y}{x}$ check quadrant

(ex.) Convert Polar $(4, 3\pi)$ to Cartesian

$x = 4\cos 3\pi = -4$

$y = 4\sin 3\pi = 0$

(ex.) Convert Cartesian $(-2, 3)$ to Polar



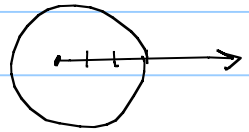
$r^2 = 4 + 9 \qquad \tan\theta = -3/2$

$r = \pm\sqrt{13} \qquad \theta \approx -.982 \text{ or } -.982 + \pi$

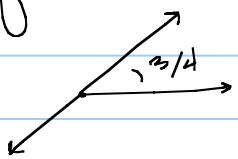
$(-\sqrt{13}, \tan^{-1}(-3/2))$ and $(\sqrt{13}, \tan^{-1}(-3/2) + \pi)$

Graphing Polar Equations

(ex) graph polar eqn $r = 3$



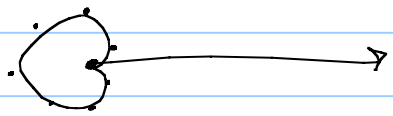
(ex) graph $\theta = 3/4$



(ex) $r = 1 - \cos\theta$

θ	r
0	0
$\pi/4$	.29289
$\pi/2$	1
$3\pi/4$	1.7071
π	2

θ	r
$5\pi/4$	1.7071
$3\pi/2$	1
$7\pi/4$	.29289
2π	0



Graphing on calculator

Go into **mode** change **function** to **Polar**

may need to set window
be careful → screen is rectangular

Tangents.

To find a tangent line to polar curve $r = f(\theta)$ recall,

$$x = r\cos\theta \Rightarrow \frac{dx}{d\theta} = -r\sin\theta + \cos\theta \frac{dr}{d\theta}$$

$$y = r\sin\theta \Rightarrow \frac{dy}{d\theta} = r\cos\theta + \sin\theta \frac{dr}{d\theta}$$

So $$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{\frac{dr}{d\theta}\sin\theta + r\cos\theta}{\frac{dr}{d\theta}\cos\theta - r\sin\theta}$$

(ex) find slope of the tangent of $r = 2 - \sin\theta$ at $\theta = \pi/3$

$$x = (2 - \sin\theta)\cos\theta$$
$$= 2\cos\theta - \cos\theta\sin\theta \Rightarrow \frac{dx}{d\theta} = 2\sin^2\theta - 2\sin\theta - 1$$

$$y = (2 - \sin\theta)\sin\theta$$
$$= 2\sin\theta - \sin^2\theta \Rightarrow \frac{dy}{d\theta} = 2\cos\theta(1 - \sin\theta)$$

$$\frac{dy}{dx} = \frac{2\cos\theta(1 - \sin\theta)}{2\sin^2\theta - 2\sin\theta - 1}$$

at $\theta = \pi/3$

$$\frac{dy}{dx} = \frac{2 - \sqrt{3}}{1 - 2\sqrt{3}} \approx -.1087$$

Horizontal tangents $\Rightarrow \frac{dy}{d\theta} = 0$

since $\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$

Vertical tangents $\Rightarrow \frac{dx}{d\theta} = 0$

since $\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$