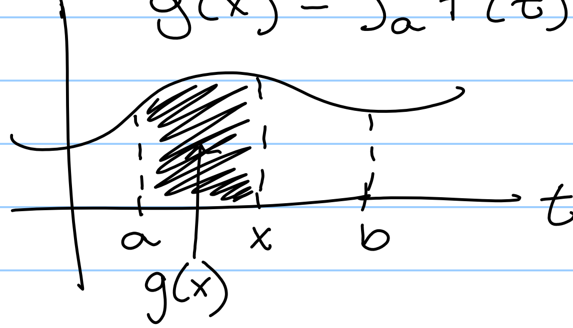
5.4

Note Title 1/5/2008

Remember that $\int_a^b f(t)\,dt$ can be interpreted as the area under the curve between a and b.

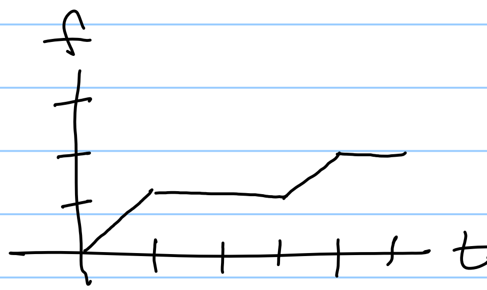
We can define a function $g(x)$ as

$$g(x) = \int_a^x f(t)\,dt, \quad a \le x \le b$$



"area so far"

ex



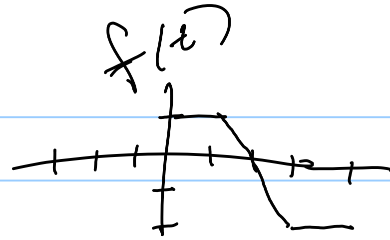
Let $g(x) = \int_0^x f(t)\,dt$

find $g(1)$? $g(1) = \int_0^1 f(t)\,dt$ = area under curve from 0 to 1

$= \frac{1}{2}$

find $g(3) = \int_0^3 f(t)\,dt = \int_0^1 f(t)\,dt + \int_1^3 f(t)\,dt$

$= \frac{1}{2} + 2 = 2.5$

ex.  let $g(x) = \int_0^x f(t)\,dt$

find $g(4) = \int_0^4 f(t)\,dt$

$= 1 + \frac{1}{2} - 1 - 2$

$= -1.5$

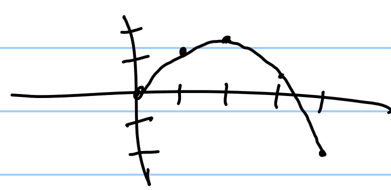
notice when $f(t)$ is above t-axis $g(x)$ is increasing (adding more area) when $f(t)$ falls below axis $g(x)$ decreases (subtracting area)

sketch $g(x)$.

$g(0)=0 \quad g(1)=1 \quad g(2)=1.5 \quad g(3)=.5$

$g(x)$ increasing $(0,2)$ dec $(2,4)$

max at 2



Fundamental Thm of Calculus Part 1

Let f be continuous on $[a,b]$

If $g(x) = \int_a^x f(t)\,dt$ then $g'(x) = f(x)$

aka $\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$

ex find derivative of $g(x) = \int_3^x \cos t^2\,dt$

$g'(x) = \cos x^2$

Sometimes our limits of integration aren't as "nice" then we need chain rule.

ex find $\frac{d}{dx}\int_1^{x^2} \sin t\,dt$

let $u = x^2$ then $\frac{d}{du}\int_1^u \sin t\,dt = \sin u$
by FTC1 $= \sin x^2$

but we want $\frac{d}{dx} = \frac{d}{du} \cdot \frac{du}{dx}$

$\sin(x^2) \cdot 2x$

$\frac{d}{dx}\int_1^{x^2} \sin t\,dt = 2x \sin x^2$

Fundamental Thm of Calculus Part 2

Let f be continuous on [a,b]

$\int_a^b f(x)\,dx = F(b) - F(a)$ where F is any antiderivative of f.

ex) Let $g(x) = \int_0^x t^2 + t\,dt$ find $g'(x)$

$g(x) = \frac{1}{3}t^3 + \frac{1}{2}t^2\Big]_0^x = \frac{1}{3}x^3 + \frac{1}{2}x^2 - 0$

$\Rightarrow g'(x) = x^2 + x$