5.5

Note Title 1/8/2008

The Substitution Rule

If $u = g(x)$ is differentiable & continuous on an interval then

$$\int f(u)\cdot u'\,dx = \int f(u)\,du$$

ex. $\int 3x^2(x^3+5)^9\,dx$

let $u = x^3+5$ then $\frac{du}{dx} = 3x^2$

$\Rightarrow du = 3x^2\,dx$

$$\int 3x^2(x^3+5)^9\,dx = \int 3x^2 u^9\,dx$$

$$= \int u^9 3x^2\,dx$$

$$= \int u^9\,du$$

$$= \frac{1}{10}u^{10}$$

$$= \frac{1}{10}(x^3+5)^{10} + C$$

check

$$\frac{d}{dx}\frac{1}{10}(x^3+5)^{10} = (x^3+5)^9\cdot 3x^2 \checkmark$$

ex. $\int (5x+3)^2\,dx$ let $u = 5x+3$ $\frac{du}{dx} = 5$

$du = 5dx$

$$\int u^2\,dx$$

$$\int \frac{1}{5}u^2 5\,dx = \int \frac{1}{5}u^2\,du = \frac{1}{15}(5x+3)^3 + C$$

check $\frac{d}{dx}\frac{1}{15}(5x+3)^3 = \frac{1}{\cancel{15}_3}(5x+3)^2\cdot\cancel{5}$ ✓

Choose u carefully. You want u' in your original.

(ex.) $\int \frac{\cos(\pi/x)}{x^2}\,dx$

<u>options for u</u>

$u = x^2 \Rightarrow du = 2x\,dx$

$u = 1/x^2 \Rightarrow du = -2/x^3\,dx$

$u = \pi/x \Rightarrow du = -\pi/x^2\,dx$ ★

since original integral has a $\frac{1}{x^2}$ in it I'll choose $u = \pi/x \Rightarrow du = -\pi/x^2\,dx$

$$\int \frac{1}{x^2}\cdot\cos\left(\frac{\pi}{x}\right)dx = \int \frac{1}{x^2}\cos u\,dx$$

$$= \int \cos u\,\frac{1}{x^2}\,dx = \int \cos u\,\frac{1}{-\pi}\,\frac{-\pi}{x^2}\,dx$$

$$= \frac{-1}{\pi}\int \cos u\,du = -\frac{1}{\pi}\sin\left(\frac{\pi}{x}\right) + C$$

Definite Integrals

Let g' be continuous on $[a,b]$ & f on $[g(a), g(b)]$ and $u = g(x)$ then

$$\int_a^b f(g(x))\, u'\, dx = \int_{g(a)}^{g(b)} f(u)\, du$$

ex) $\int_0^{\sqrt{\pi}} x \cos(x^2)\, dx$

Let $u = x^2 \Rightarrow du = 2x\, dx$

$$\int_0^{\sqrt{\pi}} \cos u \tfrac{1}{2} 2x\, dx = \tfrac{1}{2}\int_0^{\sqrt{\pi}} \cos u\, du$$

$$= \tfrac{1}{2}\int_0^{\pi} \cos u\, du = \tfrac{1}{2}\sin\pi - \tfrac{1}{2}\sin 0 = 0$$

OR

$$\tfrac{1}{2}\int_{x=0}^{x=\sqrt{\pi}} \cos u\, du = \tfrac{1}{2}\sin u\Big]_{x=0}^{x=\sqrt{\pi}} = \tfrac{1}{2}\sin x^2\Big]_0^{\sqrt{\pi}}$$

$$= \tfrac{1}{2}\sin\pi - \tfrac{1}{2}\sin 0 = 0$$