5.6

Note Title 1/8/2008

note: u-substitution undoes chain rule.
integration by parts undoes product rule

Product Rule says
$\frac{d}{dx} f\cdot g = f\cdot g' + g\cdot f'$
integrate both sides $\int \frac{d}{dx} f\cdot g = \int fg' + gf'$
$\Rightarrow f\cdot g = \int fg' + \int gf'$
$\Rightarrow \int fg' = f\cdot g - \int gf'$

Integration by parts

$$\int u\,dv = uv - \int v\,du$$

to use IbP we need to choose u & dv from our original integration then compute du & v.

ex. $\int x^5 \ln x\,dx$ let $u = \ln x$ & $dv = x^5 dx$
then $du = \frac{1}{x}dx$ & $v = \frac{1}{6}x^6$

$$\int x^5 \ln x\,dx = \ln x \cdot \frac{1}{6}x^6 - \int \frac{1}{6}x^6 \cdot \frac{1}{x}\,dx$$
$$= \frac{x^6 \ln x}{6} - \frac{1}{6}\int x^5\,dx$$
$$= \frac{1}{6}\left(x^6 \ln x - \frac{1}{6}x^6\right) + C$$

check: $\frac{d}{dx}\left[\frac{x^6\ln x}{6} - \frac{x^6}{36}\right] = \frac{1}{6}\left(x^6\frac{1}{x} + \ln x\cdot 6x^5\right) - \frac{1}{6}x^5$

$$= \frac{x^5}{6} + x^5\ln x - \frac{x^5}{6}$$

$$= x^5\ln x \quad \checkmark$$

Careful of choosing u & dv

if we'd done

$u = x^5$ & $dv = \ln x\,dx$

$\Rightarrow du = 5x^4dx$ & $v = x\ln x - x$ ← too messy!

we want u easy to differentiate & dv easy to integrate.

Once is not always enough!

ex $\int x^3e^x dx$ let $u = x^3$ and $dv = e^x$

then $du = 3x^2dx$ and $v = e^x$

by Ibp

$$\int x^3e^x dx = x^3\cdot e^x - \int e^x 3x^2 dx$$

↑ integrate by parts on just this piece.

$$\int x^3 e^x dx = x^3 e^x - \int e^x 3x^2 dx$$ let $u = 3x^2$ $dv = e^x dx$
$du = 6x\,dx$ $v = e^x$

$$= x^3 e^x - 3x^2 e^x + \int e^x 6x$$

$\uparrow$
again w/ $u = 6x$ $dv = e^x dx$
$du = 6$ $v = e^x$

$$= x^3 e^x - 3x^2 e^x + 6x e^x - \int e^x 6$$
$$= x^3 e^x - 3x^2 e^x + 6x e^x - 6e^x$$
$$= e^x (x^3 - 3x^2 + 6x - 6) + C$$

Definite Integrals by Parts

$$\int_a^b u\,dv = uv\Big]_a^b - \int_a^b v\,du$$

ex) $\int_1^4 \sqrt{x} \ln x \, dx$ let $u = \ln x$ $dv = \sqrt{x}$
$du = \frac{1}{x} dx$ $v = \frac{2}{3} x^{3/2}$

$$\int_1^4 \sqrt{x} \ln x \, dx = \left[\ln x \cdot \frac{2}{3} x^{3/2}\right]_1^4 - \int_1^4 \frac{2}{3} x^{3/2} \cdot \frac{1}{x} dx$$
$$= \frac{16}{3} \ln 4 - \frac{2}{3} \ln 1 - \frac{2}{3} \int_1^4 x^{1/2}$$
$$= \frac{16}{3} \ln 4 - \frac{2}{3} \left[\frac{2}{3} x^{3/2}\right]_1^4 = \frac{16}{3} \ln 4 - \frac{28}{9}$$

ex $\int e^{-\theta}\cos 2\theta \, d\theta$ let $u=\cos 2\theta$ & $dv=e^{-\theta}d\theta$

$du=-2\sin 2\theta$ & $v=-e^{-\theta}$

using "u-subs"

$\int e^{-\theta}d\theta$ let $u=-\theta$

$du=-1d\theta$

$-\int e^{u}du$

$=-e^{u}=-e^{-\theta}$

$$\int e^{-\theta}\cos 2\theta \, d\theta=-e^{-\theta}\cos 2\theta-\int e^{-\theta}2\sin 2\theta \, d\theta$$

Ibp again w/

$u=2\sin 2\theta$ & $dv=e^{-\theta}$

$du=4\cos 2\theta$ & $v=-e^{-\theta}$

$$\int e^{-\theta}2\sin 2\theta \, d\theta=-2e^{-\theta}\sin 2\theta+4\int e^{-\theta}\cos 2\theta \, d\theta$$

substitute this in

$$\int e^{-\theta}\cos 2\theta \, d\theta=-e^{-\theta}\cos 2\theta+2e^{-\theta}\sin 2\theta-4\int e^{-\theta}\cos 2\theta \, d\theta$$

$$\Rightarrow 5\int e^{-\theta}\cos 2\theta \, d\theta=-e^{-\theta}\cos 2\theta+2e^{-\theta}\sin 2\theta$$

$$\Rightarrow \int e^{-\theta}\cos 2\theta \, d\theta=-\frac{1}{5}e^{-\theta}\cos 2\theta+\frac{2}{5}e^{-\theta}\sin 2\theta+C$$

$\blacksquare$

hw hint: "reduction formula" is formula #7 on pg 397