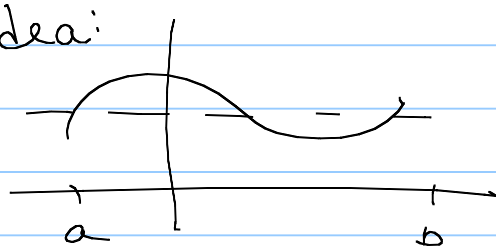
6.4

Note Title 1/30/2008

Average Value of a function

idea:



if we take a finite # of heights then

$$avg = \frac{f(x_1)+f(x_2)+\cdots+f(x_n)}{n}$$

since $\Delta x = \frac{b-a}{n} \Rightarrow n = \frac{b-a}{\Delta x}$

$$\Rightarrow avg = \frac{f(x_1)+f(x_2)+\cdots+f(x_n)}{\frac{b-a}{\Delta x}}$$

$$avg = \frac{1}{b-a}\sum_{i=1}^{n} f(x_i)\Delta x$$

if we let $n \to \infty$ we get

$$avg = \frac{1}{b-a}\lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\,\Delta x$$

$$= \frac{1}{b-a}\int_a^b f(x)\,dx$$

$$f_{avg} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

ex) find average value of $f(x) = x^2 + 1$ on $(0,3)$

$$f_{avg} = \frac{1}{3}\int_0^3 x^2 + 1 \, dx$$

$$= \frac{1}{3}\left[\frac{1}{3}x^3 + x\right]_0^3 = \frac{1}{3}(9+3-0)$$

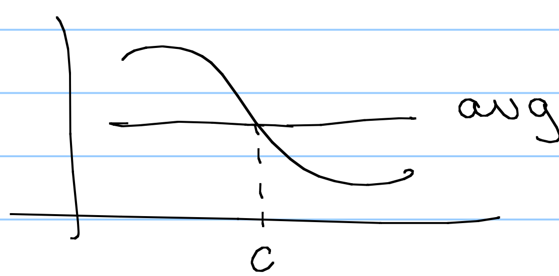
$$= 4$$

Mean Value Thm for Integrals

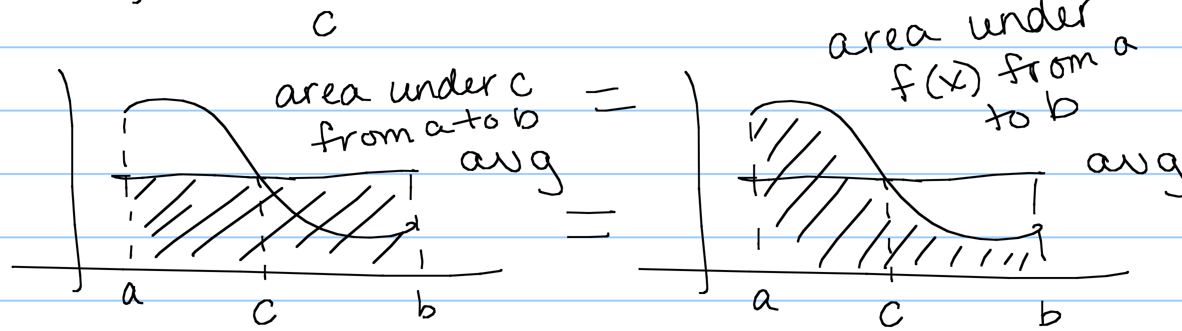
If f is continuous on $[a,b]$ then there exists a number $c \in [a,b]$ such that

$$f(c) = f_{avg} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

idea:



or



Ex. find c for the last example.

$f_{avg} = 4$

$f(c) = 4$

$c^2 + 1 = 4$

$c^2 = 3$

$c = \sqrt{3}$ since $-\sqrt{3} \notin (0,3)$