6.7

Note Title 2/1/2008

Probability

Continuous random variables take on values in an interval

ex. hight and weight

Probability density function, pdf
the function f:

$$P(a \leq X \leq b) = \int_a^b f(x)\,dx$$

for f(x) to be a pdf the following must be true

$f(x) \geq 0$ and $\int_{-\infty}^{\infty} f(x)\,dx = 1$

since Probabilities range from 0 to 1.

mean of any PDF is

$$\mu = \int_{-\infty}^{\infty} x f(x)\,dx$$

median of a pdf is the # m:

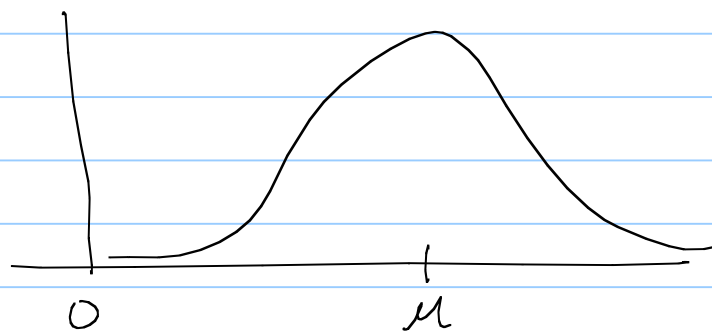
$$\int_m^{\infty} f(x)\,dx = \tfrac{1}{2}$$

Normal Distribution

the pdf of a normally distributed random variable is

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/(2\sigma^2)}$$

where μ is the mean and σ is the standard deviation



ex Let $f(x) = \begin{cases} 12x^2(1-x) & \text{if } 0 \le x \le 1 \\ 0 & \text{if } x<0 \text{ or } x>1 \end{cases}$

a. Show that $f(x)$ is a pdf

<u>Step 1</u>: show $f(x) \ge 0 \ \forall x$

Clearly $f(x) = 0 \ge 0$ for $x<0$ or $x>1$

for $0 \le x \le 1$

$12x^2 \ge 0$ and $(1-x) \ge 0$ since $0 \le x \le 1$

$\Rightarrow 12x^2(1-x) \ge 0$ for $0 \le x \le 1$

$\therefore f(x) \ge 0 \ \forall x$

<u>step 2</u>: Show $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$$\int_{-\infty}^{\infty} f(x)\,dx = \int_{-\infty}^{0} f(x)\,dx + \int_{0}^{1} f(x)\,dx + \int_{1}^{\infty} f(x)\,dx$$

$$= \int_{-\infty}^{0} 0\,dx + \int_{0}^{1} 12x^2(1-x)\,dx + \int_{1}^{\infty} 0\,dx$$

$$= 0 + \int_{0}^{1} 12x^2(1-x)\,dx + 0$$

$$= \int_{0}^{1} 12x^2(1-x)\,dx \qquad \text{let } u = 1-x$$

$$= (1-x)4x^3\Big|_0^1 + \int_0^1 4x^3\,dx \qquad du = -1dx$$

$$= (1-x)4x^3\Big|_0^1 + x^4\Big|_0^1 \qquad dv = 12x^2$$

$$= (0-0) + x^4\Big|_0^1 \qquad v = 4x^3$$

$$= 0 + (1-0) = 1$$

$$\therefore \int_{-\infty}^{\infty} f(x)\,dx = 1$$

$\Rightarrow f(x)$ is a Pdf

b. find $P(X \geq 1/2)$

$$= \int_{1/2}^{\infty} f(x)\,dx = \int_{1/2}^{1} 12x^2(1-x)\,dx + \int_{1}^{\infty} 0\,dx$$

$$= (1-x)4x^3\Big|_{1/2}^{1} + x^4\Big|_{1/2}^{1}$$

$$= \left(0 - \tfrac{1}{2}\cdot\tfrac{1}{2}\right) + \left(1 - \tfrac{1}{16}\right)$$

$$= -\tfrac{4}{16} + \tfrac{15}{16} = \tfrac{11}{16}$$

c. find mean

$$\mu = \int_{-\infty}^{\infty} x f(x)\,dx = \int_{-\infty}^{0} 0\,dx + \int_0^1 12x^3(1-x)\,dx + \int_1^{\infty} 0\,dx$$

$$= \int_0^1 12x^3(1-x)\,dx \qquad \text{Let } dv = 12x^3\,dx \quad u = 1-x$$

$$v = 3x^4 \quad du = -dx$$

$$= (1-x)3x^4 \Big|_0^1 + \int_0^1 3x^4\,dx$$

$$= (0-0) + \frac{3}{5}x^5 \Big|_0^1 = 3/5$$

ex. #10 pg 492

Heights of males in U.S. are normally distributed w/ mean of 69 inches & standard deviation of 2.8 inches.

a. find probability a man is between 65 & 73 inches?

$$P(65 < X < 73) = \int_{65}^{73} \frac{1}{2.8\sqrt{2\pi}} e^{-\frac{(x-69)^2}{(2 \cdot 2.8^2)}}$$