7.1

Note Title 2/25/2008

Say we want to model the growth of a population. The more "people" there are the faster the population will grow (more people having more kids)

Let t = time (independent variable)
P = # of people in population.

So rate of change of population is $\frac{dP}{dt}$

It can be shown for some populations (bacteria, some animals, etc) that under ideal conditions (enough food, space to live, etc)

$$\frac{dP}{dt} = k \cdot P \quad \text{or} \quad P' = k \cdot P$$

This is called <u>differential equation</u> because it's an eqn involving an unknown eqn P and it's derivative dP/dt.

A solution to $\frac{dP}{dt} = k \cdot P$ is the family
$P(t) = Ce^{kt}$ for any constant C
because
$P'(t) = Ce^{kt} \cdot K$
Plugging into
$P' = k \cdot P$
we get
$Ce^{kt} \cdot K = K \cdot Ce^{kt}$ so it checks.

<u>family of curves</u> - a group of curves that have same general definition.

<u>Order of dif. eq.</u> - order of the highest derivative in eqn.

<u>Solution of a dif. eq</u> - is a function $f(x)$ that satisfies the dif. eq.

<u>initial condition/initial-value problems</u>
you're given a point $y(t_0) = y_0$ to distinguish between the members of a family of curves (a.k.a. to find constants)

it is assumed that y is a function of x.

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ex. Lets look at dif. eq. $y' = xy^3$

When $x \to 0$, $y' \to 0 \Rightarrow$ horizontal tang at $x=0$

when $x \to \infty$, $y' \to \infty$

Show $y = (c-x^2)^{-1/2}$ are solns to dif. eq.

if $y = (c-x^2)^{-1/2}$ then $y' = x(c-x^2)^{-3/2}$

substituting gives

$$x(c-x^2)^{-3/2} = x((c-x^2)^{-1/2})^3$$
$$= x(c-x^2)^{-3/2} \checkmark$$

To find initial-value solution to

$y(0)=2$

$y(0) = (c-0^2)^{-1/2} = 2$

$\Rightarrow c^{-1/2} = 2$

$\Rightarrow c = 1/4$

so $y = (1/4 - x^2)^{-1/2}$