7.3

Note Title 2/27/2008

Seperable Equations

If we can rewrite a diff. Eq. as $\frac{dy}{dx} = f(x)g(y)$ then we can write

$$g(y)\,dy = f(x)\,dx$$

then we can integrate both sides

$$\int g(y)\,dy = \int f(x)\,dx$$

(ex) let $\frac{dy}{dx} = -\frac{x}{y}$ then $y\,dy = -x\,dx$

$$\Rightarrow \int y\,dy = \int -x\,dx$$

$$\tfrac{1}{2}y^2 + C_1 = -\tfrac{1}{2}x^2 + C_2$$

$$y^2 + x^2 = K \quad \text{where } K = 2(C_2 - C_1)$$

Notice this is a circle with radius $\sqrt{K}$, centered at origin.

(family of curves)

Initial value problems.

What if we want to know the exact circle (aka the radius). If we know one point on the circle then we can figure it out. This point is initial value.

initial-value

(ex.) $\frac{dw}{d\theta} = \theta w^2 \sin\theta^2$, $w(0)=1$

$\frac{1}{w^2}dw = \theta \sin\theta^2 d\theta$

$\int w^{-2}dw = \int \theta \sin\theta^2 d\theta$ let $u=\theta^2, \frac{1}{2}du = \theta d\theta$

$-w^{-1} = -\frac{1}{2}\cos u + C$

$w^{-1} = \frac{1}{2}\cos\theta^2 + C$

from above $w(0)=1$ when $\theta=0$ $w=1$

$1^{-1} = \frac{1}{2}\cos(0^2) + C$

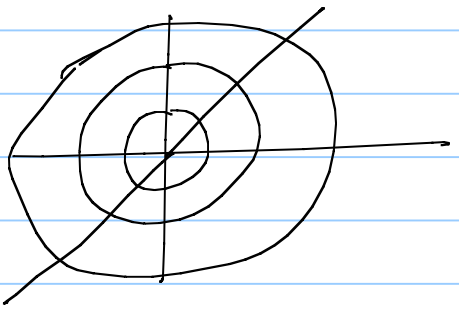
$1 = \frac{1}{2} + C$

$\frac{1}{2} = C$

$\frac{1}{w} = \frac{1}{2}\cos\theta^2 + \frac{1}{2} \Rightarrow w = \frac{2}{\cos\theta^2 + 1}$

Orthogonal Trajectory of a family of curves is a curve that intersects each curve in the family at right angles.

(ex.)



(ex) find orthogonal trajectory of $y^2 = Kx^3$

$y^2 = Kx^3 \Rightarrow 2y\frac{dy}{dx} = 3Kx^2$

$\Rightarrow \frac{dy}{dx} = \frac{3Kx^2}{2y}$

So ortho. traj. has

$$\frac{dy}{dx} = -\frac{2y}{3Kx^2}$$

we don't want the K in there so solve $y^2 = Kx^3$ for K

$K = y^2/x^3$

$\Leftrightarrow$ substitute

$$\frac{dy}{dx} = -\frac{2y}{3\left(\frac{y^2}{x^3}\right)x^2} = -\frac{2x}{3y}$$

This diff. Eq is seperable

$$\frac{dy}{dx} = -\frac{2x}{3y} \Rightarrow 3y\,dy = -2x\,dx$$

$$\int 3y\,dy = \int -2x\,dx$$

$$\frac{3}{2}y^2 = -x^2 + C \Rightarrow \frac{3}{2}y^2 + x^2 = C$$

Mixing Problems

(ex.) The air in a room with volume 180 m^3 contains .15% carbon dioxide initially. Fresher air with only .05% carbon dioxide flows into the room at 2 m^3/min & the mixed air flows out at same rate. Find equation for percent of carbon dioxide at time t.

C = amt carbon $c(0) = .0015 * 180 = .27\ m^3$

$\frac{dC}{dt}$ = rate in − rate out

rate in: $.0005 * 2\ m^3/min = .001$

rate out: $\frac{C}{180\ m^3} * 2\ m^3/min = C/90$

$\frac{dC}{dt} = .001 - C/90$

$= \frac{.09 - C}{90}$

seperable

$\int \frac{1}{.09 - C}\, dC = \int \frac{1}{90}\, dt$

$-\ln|.09 - C| = \frac{t}{90} + K$

At $t = 0$, $C = .27 \Rightarrow K = -\ln .18$

$-\ln|.09 - C| = \frac{t}{90} - \ln .18$

so LVL too hard to solve for C.