7.6

Note Title 3/3/2008

Predator-Prey Systems

We now talk about environments where there are 2 species (predators & prey) We assume prey have ideal conditions.

We usually use $R(t)$ ← prey (for rabbits) and $W(t)$ ← predator (for wolves)

If there was no predator

$$\frac{dR}{dt} = k \cdot R$$

If there was no prey

$$\frac{dW}{dt} = -kW$$ (wolves would die out w/ no rabbits)

When both species are present we need to make some assumptions

- prey die by being eaten by predator
- birth & survival of predator depend on prey.
- rate that species encounter each other is proportional to RW (the more of each the more they find each other)

This gives

$$\frac{dR}{dt} = kR - aRW \qquad \frac{dW}{dt} = -rW + bRW$$

where a, b, k, r are positive constants

these are called Predator-Prey or Lotka-Volterra Equations

Unfortunately, it is usually impossible to find explicit formulas for solutions. We can however get some general info.

(8.) Suppose rabbits & wolves fit Lotka-Volterra eqns above with $k=.08$ $a=.001$ $r=.02$ $b=.00002$. t measured in months.

a) find constant solutions

$$\frac{dR}{dt} = .08R - .001RW$$

$$\frac{dW}{dt} = -.02W + .00002RW$$

If R and W are constants then $R'=0$ & $W'=0$

$$R(.08-.001W)=0$$
$$W(-.02+.00002R)=0$$

Ignore $R=0, W=0$ (no rabbits, no wolves)

$$.08 = .001W \Rightarrow W = 80$$
$$.02 = .00002R \quad R = 1000$$

1000 rabbits is exactly enough to sustain 80 wolves. Wolves/rabbits will never increase/decrease.

b) find $\frac{dW}{dR}$?

$$\frac{dW}{dR} = \frac{\frac{dW}{dt}}{\frac{dR}{dt}} = \frac{-.02W + .00002RW}{.08R - .001RW}$$

c) Draw a direction field for $\frac{dW}{dR}$.
Note equilibrium point (1000, 80)

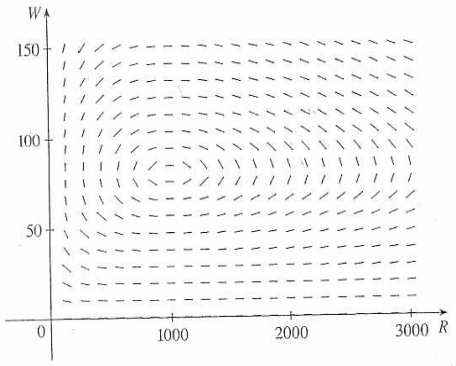
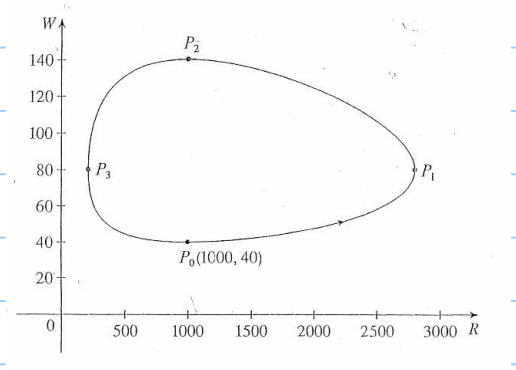


FIGURE 1 Direction field for the predator-prey system

called phase plane. Solution curves are called phase trajectories

d) Suppose at some time there are 1000 rabbits & 40 wolves. Draw phase trajectory & describe changes in both populations



"Start at (1000, 40)

$\frac{dR}{dt} = .08(1000) - .001(1000)(40) = 40 > 0$

so rabbits are increasing ⇒ move counterclockwise

As time goes by # of rabbits increase ⇒ wolves increase until about 2800 rabbits & 80 wolves.

Then the wolves start to take over the rabbits so R decreases, W still increases until about (1000, 140) then too few R to feed W so W decreases.

Then too few W to kill off R so R starts to increase

⋮

e) Sketch R and W using part d.

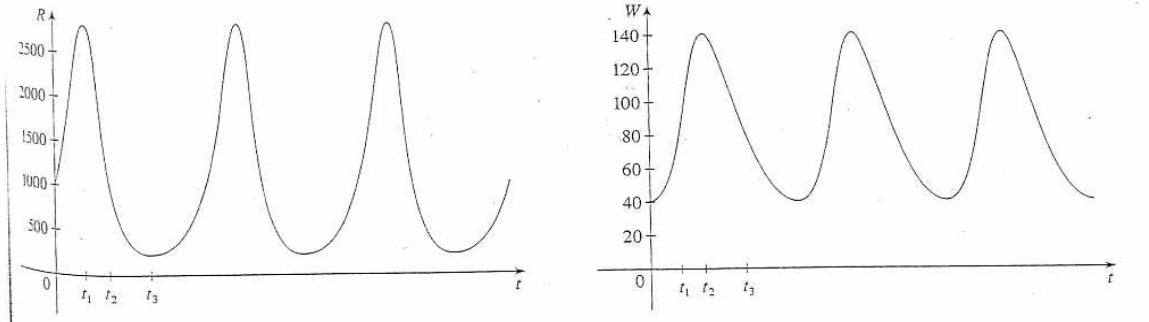


FIGURE 4 Graphs of the rabbit and wolf populations as functions of time

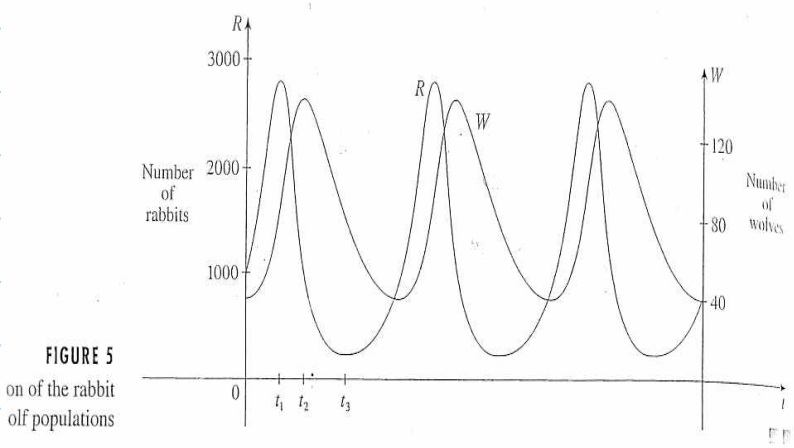


FIGURE 5
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olf populations