8.2

Note Title 3/17/2008

Series

$$a_1 + a_2 + a_3 + \cdots + a_n + \cdots = \sum a_n$$

Some series add up to a number, some don't.

ex. $\sum n = 1+2+3+4+5+\cdots \longrightarrow \infty$

ex. $\sum \frac{1}{2^n} = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \cdots = 1$

But how did I know $\sum \frac{1}{2^n} = 1$

Partial sums

	ex
$S_1 = a_1$	$1/2$
$S_2 = a_1 + a_2$	$1/2 + 1/4 = .75$
$S_3 = a_1 + a_2 + a_3$	$1/2 + 1/4 + 1/8 = .875$
$\vdots$	$= .9375$
$S_n = a_1 + a_2 + \cdots + a_n = \sum_{i=1}^{n} a_i$	$= .96875$
	$\vdots$

If the new sequence $\{S_n\}$ converges then $\lim_{n\to\infty} S_n = S$ then the series $\sum a_n$ converges and $\sum a_n = S$. ← s is called sum of series

If $\{S_n\}$ diverges then $\sum a_n$ diverges.

ex. $\sum \frac{1}{2^n}$ converges. The sum of the series is 1.

Harmonic Series $\sum \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots$
diverges.

Geometric Series

$\sum ar^{n-1} \longrightarrow$ each term is the previous term multiplied by r

ex. $\sum 5 \cdot 2^{n-1} = 5 + 10 + 20 + 40 + 80 + 160 + \cdots$
$\sum 2 \cdot 3^{n-1} = 2 + 6 + 18 + 54 + \cdots$
$\sum 2 \cdot (\frac{1}{2})^{n-1} = 2 + 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots$

Assume $r \neq 1$ (since that's kinda boring)
$S_n = a + ar + ar^2 + ar^3 + ar^4 + \cdots + ar^{n-1}$
$rS_n = \quad ar + ar^2 + ar^3 + ar^4 + \cdots + ar^{n-1} + ar^n$
Subtract these equations
$S_n - rS_n = a - ar^n$
$S_n(1-r) = a(1-r^n)$

$$S_n = \frac{a(1-r^n)}{(1-r)}$$

If $-1 < r < 1$ then $\lim_{n\to\infty} r^n = 0$ (from previous section)
$$\lim_{n\to\infty} S_n = \lim_{n\to\infty} \frac{a(1-r^n)}{(1-r)} = \frac{a}{1-r} \lim_{n\to\infty} 1 - r^n$$
$$= \frac{a}{1-r}(1-0) = \frac{a}{1-r}$$
So if $-1 < r < 1$ then $\sum ar^{n-1} = \frac{a}{1-r}$

If $r > 1$ or $r < -1$ then $\{r^n\}$ is divergent
$\Rightarrow \sum ar^{n-1}$ is divergent.

Geometric Series is convergent &

$\sum_{n=1}^{\infty} ar^{n-1} = \frac{a}{1-r}$ if $|r|<1$ $\left[\text{or } \sum_{n=0}^{\infty} ar^{n} = \frac{a}{1-r} \text{ if } |r|<1\right]$

If $|r| \geq 1$ the geometric series diverges.

ex. find $\sum \frac{3}{2^{n-1}}$

$= \sum 3\left(\frac{1}{2}\right)^{n-1}$ ← geometric w/ $a=3, r=\frac{1}{2}$

$= \frac{3}{1-\frac{1}{2}} = 6$

ex. find $\sum \left(\frac{3}{2}\right)^{n-1}$ now $a=1$, $r=\frac{3}{2}$ so the series diverges.

ex. $.\overline{08} = \frac{8}{10^2} + \frac{8}{10^4} + \frac{8}{10^6} + \frac{8}{10^8} + \cdots$

$.08 + .0008 + .000008 + \cdots$

$= \sum_{n=0}^{\infty} \frac{8}{10^2}\left(\frac{1}{10}\right)^{2n}$

$= \sum_{n=0}^{\infty} \frac{8}{10^2}\left(\frac{1}{10^2}\right)^{n}$ geometric series w/ $a=\frac{8}{10^2}$ $r=\frac{1}{100}$

$= \frac{8/100}{1-1/100}$

$= \frac{8/100}{99/100} = 8/99$

If the series $\sum_{n=1}^{\infty} a_n$ is convergent, then
$\lim_{n\to\infty} a_n = 0$

note: the converse of this is not necessarily true. That is, just because $\lim_{n\to\infty} a_n = 0$ does NOT mean $\sum a_n$ is conv.

Test for divergence

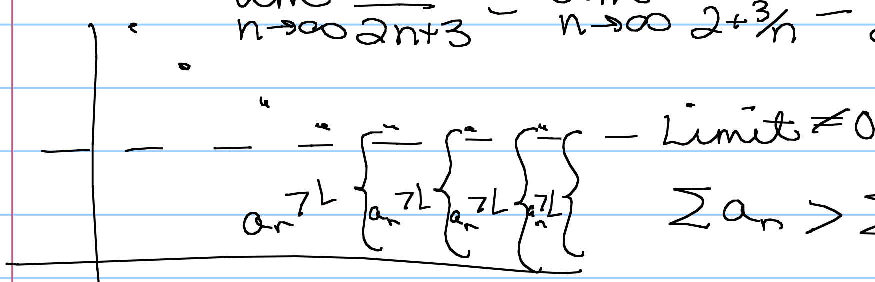
If $\lim_{n\to\infty} a_n$ dne or if $\lim_{n\to\infty} a_n \neq 0$ then $\sum a_n$ is divergent.

ex. Show that $\sum 2^n$ diverges

$\lim_{n\to\infty} 2^n = \infty \neq 0$ so $\sum 2^n$ diverges

ex. Show $\sum \frac{n}{2n+3}$ diverges

$\lim_{n\to\infty} \frac{n}{2n+3} = \lim_{n\to\infty} \frac{1}{2+3/n} = \frac{1}{2} \neq 0$ diverges



$\sum a_n > \sum L \to \infty$

If $\sum a_n$ & $\sum b_n$ are convergent (c is constant)

$\sum c a_n = c \sum a_n$ $\qquad$ $\sum a_n + b_n = \sum a_n + \sum b_n$

ditto for $-$