8.5

Note Title 3/17/2008

Power Series

$$\sum_{n=0}^{\infty} C_n x^n = C_0 + C_1 x + C_2 x^2 + C_3 x^3 + \cdots$$

where C_i's are constants, x variable.

For each fixed value of x the series is a series of constants that we can test for convergence or divergence. (may converge for some x's & diverge for others)

$f(x) = C_0 + C_1 x + C_2 x^2 + C_3 x^3 + \cdots$ is a function whose domain is the set of all x for which the series converges.

Notice if we let $C_n = 1$ $\forall n$ then we get the geometric series

$$\sum x^n = 1 + x + x^2 + x^3 + \cdots$$

which converges when $-1 < x < 1$ and diverges when $|x| \geq 1$.

Power Series in $(x-a)$ or "centered at a" or "about a"

$$\sum_{n=0}^{\infty} C_n (x-a)^n = C_0 + C_1 (x-a) + C_2 (x-a)^2 + \cdots$$

Thm

For a given power series $\sum_{n=0}^{\infty} c_n (x-a)^n$ there are 3 possibilities:

i) The series converges only when $x=a$

ii) The series converges for all x

iii) The series converges if $|x-a|<R$ and diverges if $|x-a|>R$ for some positive number R.

The #R in iii is called the radius of convergence ($R=0$ for i and $R=\infty$ for ii)

The interval of convergence is all values of x for which series converges.

ex) $\sum_{n=1}^{\infty} \frac{(-2)^n}{\sqrt{n}} (x+3)^n$ Ratio test note $(-2)^n = (-1)^n (2)^n$

$$\lim_{n\to\infty} \left| \frac{2^{n+1}}{\sqrt{n+1}} (x+3)^{n+1} \cdot \frac{\sqrt{n}}{2^n (x+3)^n} \right|$$

$$= \lim_{n\to\infty} \left| \frac{2^{n+1} (x+3)^{n+1} \sqrt{n}}{2^n (x+3)^n \sqrt{n+1}} \right| = \lim_{n\to\infty} \left| 2(x+3)\sqrt{\frac{n}{n+1}} \right|$$

$$= \lim_{n\to\infty} \left| \frac{2(x+3)}{\sqrt{\frac{n+1}{n}}} \right| = \lim_{n\to\infty} \left| \frac{2(x+3)}{\sqrt{1+1/n}} \right| = |2(x+3)|$$

$\Rightarrow \lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = 2|x+3|$ for conv. need $2|x+3|<1$

$\Rightarrow R = 1/2$ since $|x+3| < 1/2$

$$|x+3| < 1/2 \Rightarrow -1/2 < x+3 < 1/2$$
$$\Rightarrow -7/2 < x < -5/2$$

So interval of convergence is $(-7/2, -5/2)$

(d) $\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$ Ratio test

$$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left|\frac{x^{2(n+1)}}{(2(n+1))!} \cdot \frac{(2n)!}{x^{2n}}\right|$$

$$= \lim_{n\to\infty} \left|\frac{x^{2n+2}}{x^{2n}} \cdot \frac{\cancel{2^n} n!}{\cancel{2^n}(n+1)!}\right| = \lim_{n\to\infty} \left|x^2 \frac{n!}{(n+1)!}\right|$$

$$= \lim_{n\to\infty} \left|\frac{x^2}{n+1}\right| = 0 < 1$$ so series converges for all x.

how did I know $(2n)! = 2^n n!$

$$n! = n(n-1)(n-2)(n-3) \cdots 3 \cdot 2 \cdot 1$$
$$(2n)! = 2n\,2(n-1)\,2(n-2)\,2(n-3) \cdots 2 \cdot 3 \cdot 2 \cdot 2 \cdot 2 \cdot 1$$
$$= 2^n n(n-1)(n-2) \cdots 3 \cdot 2 \cdot 1$$
$$= 2^n n!$$

and $\frac{n!}{(n+1)!} = \frac{1}{n+1}$

$$\frac{n!}{(n+1)!} = \frac{\cancel{n}\cancel{(n-1)}\cancel{(n-2)} \cdots \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}{(n+1)\cancel{n}\cancel{(n-1)}\cancel{(n-2)} \cdots \cancel{3} \cdot \cancel{2} \cdot \cancel{1}} = \frac{1}{n+1}$$