8.6

Note Title 3/18/2008

Recall the geometric series

$$\sum_{n=0}^{\infty} ar^{n-1} = a + ar + ar^2$$

if $|r|<1$ then

$$\sum ar^n = \frac{a}{1-r}$$

We can use this "in reverse" to find a sum for an equation

(ex.) find power series representation for

$f(x) = \frac{1}{1+9x^2}$ $a=1$, $r=-9x^2$

$$f(x) = \sum_{n=0}^{\infty} (-9x^2)^n = (-1)^n 9^n x^{2n}$$

Converges when $|-9x^2|<1 \Rightarrow 9x^2<1$

$x^2 < 1/9 \Rightarrow -1/3 < x < 1/3$

(ex) find one for $f(x) = \frac{x}{4x+1} = \frac{x}{1+4x}$ $a=1$ $r=-4x$

have the extra x in numerator so

$$f(x) = x\sum(-4x)^n = \sum(-1)^n \cdot 4^n x^{n+1}$$

converges when $|-4x|<1 \Rightarrow 4|x|<1 \Rightarrow |x|<1/4$

$-1/4 < x < 1/4$

What is this good for?

(ex) $\int \frac{x}{4x+1}\,dx = \int \Sigma(-1)^n \cdot 4^n x^{n+1}\,dx$

$$= \int \underset{n=0}{x^1} - \underset{n=1}{4x^2} + \underset{n=2}{16x^3} - \underset{n=3}{64x^4} + \cdots$$

$$= \frac{1}{2}x^2 - \frac{4}{3}x^3 + \frac{16}{4}x^4 - \frac{64}{5}x^5 \cdots$$

$$= \sum \frac{(-1)^n 4^n x^{n+2}}{n+2}$$

$$\int \Sigma(-1)^n \cdot 4^n x^{n+1}\,dx = \Sigma \frac{(-1)^n 4^n}{n+2} x^{n+2}$$

In general,

If power series $\Sigma c_n(x-a)^n$ has radius of convergence $R>0$, then the function f:

$$f(x) = c_0 + c_1(x-a) + c_2(x-a)^2 + \cdots = \sum_{n=0}^{\infty} c_n(x-a)^n$$

has

$$f' = c_1 + 2c_2(x-a) + 3c_3(x-a)^2 + \cdots = \sum_{n=0}^{\infty} nc_n(x-a)^{n-1}$$

and

$$\int f(x)\,dx = c_0(x-a) + \frac{c_1}{2}(x-a)^2 + \frac{c_2}{3}(x-a)^3 + \cdots + \sum_{n=0}^{\infty} \frac{c_n}{n+1}(x-a)^{n+1}$$