8.8

Note Title 3/20/2008

Binomial Series

You may know

$$(a+b)^k = a^k + ka^{k-1}b + \frac{k(k-1)}{2!}a^{k-2}b^2 + \cdots + kab^{k-1} + b^k$$

$$= \sum_{n=0}^{k} \binom{k}{n} a^{k-n} b^n$$

when k is positive integer.

We can now expand this to $k \in \mathbb{R}$

Binomial Series Thm

If $k \in \mathbb{R}$ and $|x| < 1$ then

$$(1+x)^k = 1 + kx + \frac{k(k-1)}{2!}x^2 + \frac{k(k-1)(k-2)}{3!}x^3 + \cdots$$

$$= \sum_{n=0}^{\infty} \binom{k}{n} x^n$$

where $\binom{k}{n} = \frac{k(k-1)\cdots(k-n+1)}{n!}$ $n \geq 1$ & $\binom{k}{0} = 1$

on calculator

nCr

this converges when $|x| < 1$

ex. Use binomial series to expand $(1-x)^{2/3}$ as power series. State radius of convergence

$$(1-x)^{2/3} = \sum_{n=0}^{\infty} \binom{2/3}{n} (-x)^n = \sum_{n=0}^{\infty} (-1)^n \binom{2/3}{n} x^n$$

$$|-x| < 1 \Rightarrow |x| < 1 \quad \text{so } R = 1$$