

FINAL EXAM REVIEW

Math 1200 – Fall 2006

The final exam will be on Monday, Dec 18 at 9:40 in room _____

The exam will cover Chapters 2, 3, 4, 5, and 8

All homework from Chapter 8 is due at the exam (late assignments are NOT accepted).

You may use your calculator on this exam.

You may NOT use your notes, homework, book, or neighbors on this exam. You do get an $8\frac{1}{2}$ X 11 “cheat-sheet” for this exam.

Below is a review for this exam. Anything on the review could possibly be on the exam. The exam will be shorter than the review.

In Exercises 1–3, determine whether each relation is a function. Give the domain and range for each relation.

- $\{(2, 7), (3, 7), (5, 7)\}$
- $\{(1, 10), (2, 500), (13, \pi)\}$
- $\{(12, 13), (14, 15), (12, 19)\}$

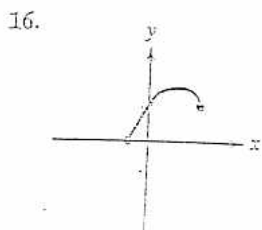
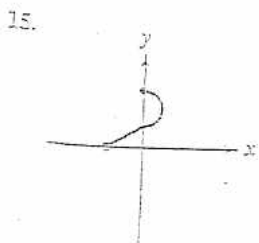
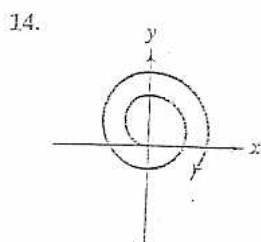
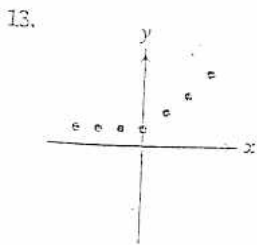
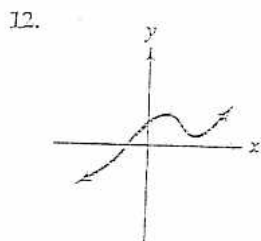
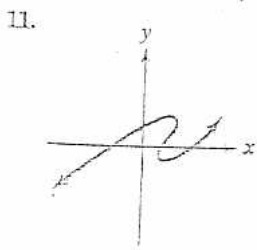
In Exercises 4–6, determine whether each equation defines y as a function of x .

- $2x + y = 8$
- $2x + y^2 = 6$
- $3x^2 + y = 14$

In Exercises 7–10, evaluate each function at the given values of the independent variable and simplify.

- $f(x) = 5 - 7x$
 - $f(4)$
 - $f(x + 3)$
 - $f(-x)$
- $g(x) = 3x^2 - 5x + 2$
 - $g(0)$
 - $g(-2)$
 - $g(x - 1)$
 - $g(-x)$
- $g(x) = \begin{cases} \sqrt{x - 4} & \text{if } x \geq 4 \\ 4 - x & \text{if } x < 4 \end{cases}$
 - $g(13)$
 - $g(0)$
 - $g(-3)$
- $f(x) = \begin{cases} \frac{x^2 - 1}{x - 1} & \text{if } x \neq 1 \\ 12 & \text{if } x = 1 \end{cases}$
 - $f(-2)$
 - $f(1)$
 - $f(2)$

In Exercises 11–16, use the vertical line test to identify graphs in which y is a function of x .



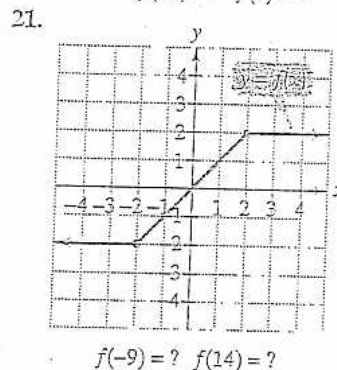
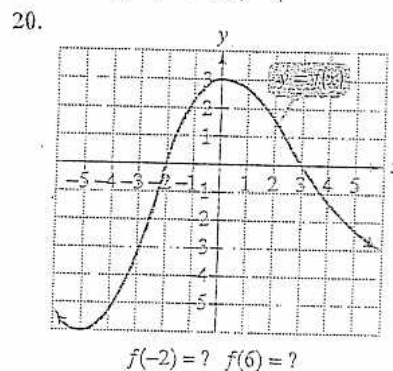
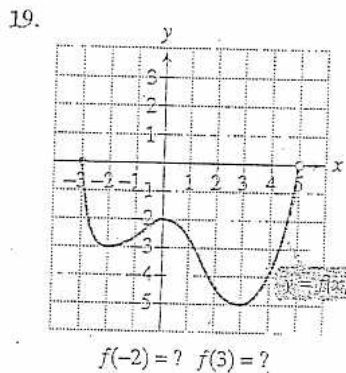
In Exercises 17–18, find and simplify the difference quotient

$$\frac{f(x+h) - f(x)}{h}, \quad h \neq 0$$

for the given function.

- $f(x) = 8x - 11$
- $f(x) = -2x^2 + x + 10$

In Exercises 19–21, use the graph to determine a. the function's domain; b. the function's range; c. the x -intercepts, if any; d. the y -intercept, if any; e. intervals on which the function is increasing, decreasing, or constant; and f. the function values indicated below the graphs.



In Exercises 22–23, find each of the following:

- The numbers, if any, at which f has a relative maximum. What are these relative maxima?
- The numbers, if any, at which f has a relative minimum. What are these relative minima?

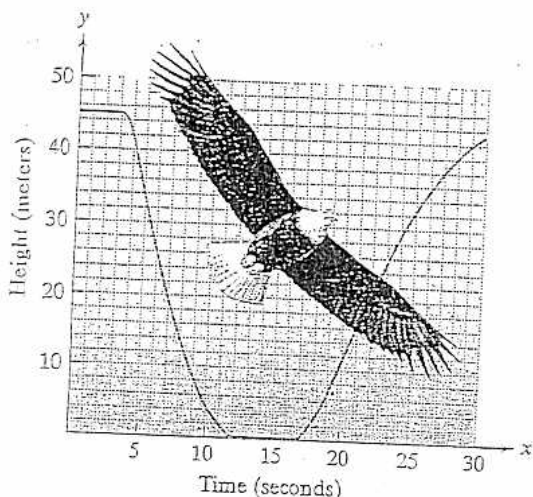
22. Use the graph in Exercise 19.

23. Use the graph in Exercise 20.

In Exercises 24–26, determine whether each function is even, odd, or neither. State each function's symmetry. If you are using a graphing utility, graph the function and verify its possible symmetry.

- $f(x) = x^3 - 5x$
- $f(x) = x^4 - 2x^2 + 1$
- $f(x) = 2x\sqrt{1 - x^2}$

27. The graph shows the height, in meters, of an eagle in terms of its time, in seconds, in flight.



- Is the eagle's height a function of time? Use the graph to explain why or why not.
 - On which interval is the function decreasing? Describe what this means in practical terms.
 - On which intervals is the function constant? What does this mean for each of these intervals?
 - On which interval is the function increasing? What does this mean?
28. A cargo service charges a flat fee of \$5 plus \$1.50 for each pound or fraction of a pound. Graph shipping cost, $C(x)$, in dollars, as a function of weight, x , in pounds, for $0 < x \leq 5$.

In Exercises 29–32, find the slope of the line passing through each pair of points or state that the slope is undefined. Then indicate whether the line through the points rises, falls, is horizontal, or is vertical.

29. $(3, 2)$ and $(5, 1)$ 30. $(-1, -2)$ and $(-3, -4)$
 31. $(-3, \frac{1}{4})$ and $(6, \frac{1}{4})$ 32. $(-2, 5)$ and $(-2, 10)$

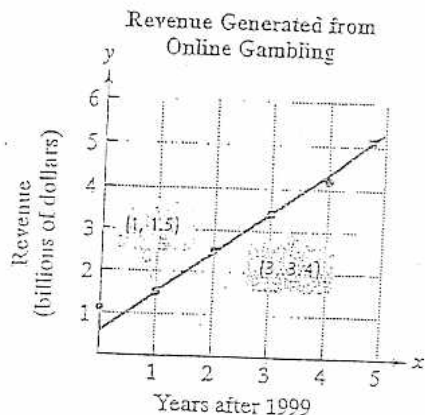
In Exercises 33–36, use the given conditions to write an equation for each line in point-slope form and slope-intercept form.

- Passing through $(-3, 2)$ with slope -6
- Passing through $(1, 6)$ and $(-1, 2)$
- Passing through $(4, -7)$ and parallel to the line whose equation is $3x + y - 9 = 0$
- Passing through $(-3, 6)$ and perpendicular to the line whose equation is $y = \frac{1}{3}x + 4$
- Write the general form of the equation of the line passing through $(-12, -1)$ and perpendicular to the line whose equation is $6x - y - 4 = 0$.

In Exercises 38–41, give the slope and y-intercept of each line whose equation is given. Then graph the line.

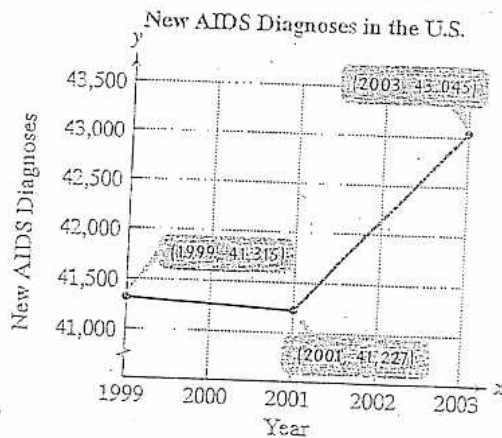
38. $y = \frac{2}{3}x - 1$ 39. $f(x) = -4x + 5$
 40. $2x + 3y + 6 = 0$ 41. $2y - 8 = 0$
 42. Graph using intercepts: $2x - 5y - 10 = 0$.
 43. Graph: $2x - 10 = 0$.

44. You can click a mouse and bet the house. The points in the graph show the dizzying growth of online gambling. With more than 1800 sites, the industry has become the Web's biggest moneymaker.



Source: Newsweek

- Use the two points whose coordinates are shown by the voice balloons to find the point-slope form of the equation of the line that models revenue from online gambling, y , in billions of dollars, x years after 1999.
 - Write the equation in part (a) in slope-intercept form.
 - In 2003, nearly \$3.5 billion was lost on Internet bets, triggering a sharp backlash that threatened to shut down Internet wagering. If this crackdown on the industry is not successful, use your slope-intercept model to predict the billions of dollars in revenue from online gambling in 2009.
45. The graph shows new AIDS diagnoses among the general U.S. population, y , for year x , where $1999 \leq x \leq 2003$.

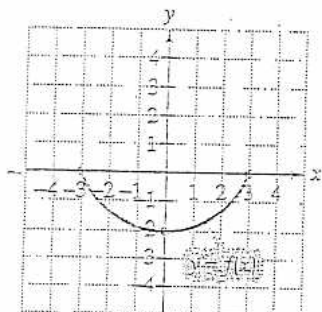


Source: Centers for Disease Control

- Find the slope of the line passing through $(1999, 41,315)$ and $(2001, 41,227)$. Then express the slope as a rate of change with the proper units attached.
- Find the slope of the line passing through $(2001, 41,227)$ and $(2003, 43,045)$. Then express the slope as a rate of change.
- Draw a line passing through $(1999, 41,315)$ and $(2003, 43,045)$ and find its slope. Is the slope the average of the slopes of the lines that you found in parts (a) and (b)? Explain your answer.

46. Find the average rate of change of $f(x) = x^2 - 4x$ from $x_1 = 5$ to $x_2 = 9$.

In Exercises 47–51, use the graph of $y = f(x)$ to graph each function g .



47. $g(x) = f(x + 2) + 3$ 48. $g(x) = \frac{1}{2}f(x - 1)$
 49. $g(x) = -f(2x)$ 50. $g(x) = 2f(\frac{1}{2}x)$
 51. $g(x) = -f(-x) - 1$

In Exercises 52–55, begin by graphing the standard quadratic function, $f(x) = x^2$. Then use transformations of this graph to graph the given function.

52. $g(x) = x^2 + 2$ 53. $h(x) = (x + 2)^2$
 54. $r(x) = -(x + 1)^2$ 55. $y(x) = \frac{1}{2}(x - 1)^2 + 1$

In Exercises 56–58, begin by graphing the square root function, $f(x) = \sqrt{x}$. Then use transformations of this graph to graph the given function.

56. $g(x) = \sqrt{x + 3}$ 57. $h(x) = \sqrt{3 - x}$
 58. $r(x) = 2\sqrt{x + 2}$

In Exercises 59–61, begin by graphing the absolute value function, $f(x) = |x|$. Then use transformations of this graph to graph the given function.

59. $g(x) = |x + 2| - 3$ 60. $h(x) = -|x - 1| + 1$
 61. $r(x) = \frac{1}{2}|x + 2|$

In Exercises 62–64, begin by graphing the standard cubic function, $f(x) = x^3$. Then use transformations of this graph to graph the given function.

62. $g(x) = \frac{1}{2}(x - 1)^3$ 63. $h(x) = -(x + 1)^3$
 64. $r(x) = \frac{1}{4}x^3 - 1$

In Exercises 65–67, begin by graphing the cube root function, $f(x) = \sqrt[3]{x}$. Then use transformations of this graph to graph the given function.

65. $g(x) = \sqrt[3]{x + 2} - 1$ 66. $h(x) = -\sqrt[3]{2x}$
 67. $r(x) = -2\sqrt[3]{-x}$

In Exercises 68–73, find the domain of each function.

68. $f(x) = x^2 + 6x - 8$ 69. $g(x) = \frac{4}{x - 7}$

70. $h(x) = \sqrt{8 - 2x}$

71. $f(x) = \frac{x}{x^2 + 4x - 21}$

72. $g(x) = \frac{\sqrt{x - 2}}{x - 5}$

73. $f(x) = \sqrt{x - 1} + \sqrt{x + 5}$

In Exercises 74–76, find $f + g$, $f - g$, $f \cdot g$, and $\frac{f}{g}$. Determine the domain for each function.

74. $f(x) = 3x - 1$, $g(x) = x - 5$
 75. $f(x) = x^2 + x + 1$, $g(x) = x^2 - 1$
 76. $f(x) = \sqrt{x + 7}$, $g(x) = \sqrt{x - 2}$

In Exercises 77–78, find a. $(f \circ g)(x)$; b. $(g \circ f)(x)$; c. $(f \circ g)(3)$.

77. $f(x) = x^2 + 3$, $g(x) = 4x - 1$
 78. $f(x) = \sqrt{x}$, $g(x) = x + 1$

In Exercises 79–80, find a. $(f \circ g)(x)$; b. the domain of $(f \circ g)$.

79. $f(x) = \frac{x + 1}{x - 2}$, $g(x) = \frac{1}{x}$
 80. $f(x) = \sqrt{x - 1}$, $g(x) = x + 3$

In Exercises 81–82, express the given function h as a composition of two functions f and g so that $h(x) = (f \circ g)(x)$.

81. $h(x) = (x^2 + 2x - 1)^4$
 82. $h(x) = \sqrt[3]{7x + 4}$

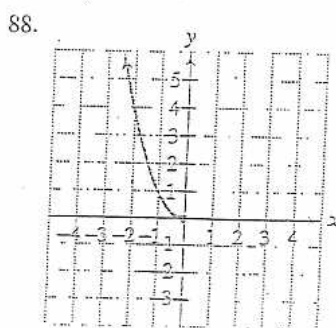
In Exercises 83–84, find $f(g(x))$ and $g(f(x))$ and determine whether each pair of functions f and g are inverses of each other.

83. $f(x) = \frac{3}{5}x + \frac{1}{2}$ and $g(x) = \frac{5}{3}x - 2$
 84. $f(x) = 2 - 5x$ and $g(x) = \frac{2 - x}{5}$

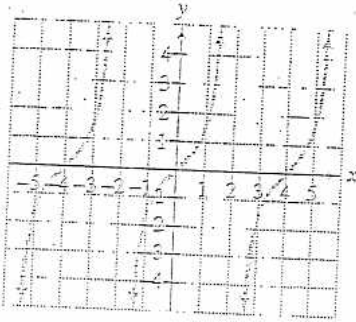
The functions in Exercises 85–87 are all one-to-one. For each function,

- a. Find an equation for $f^{-1}(x)$, the inverse function.
 b. Verify that your equation is correct by showing that $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$.
 85. $f(x) = 4x - 3$ 86. $f(x) = 8x^3 + 1$
 87. $f(x) = \frac{2}{x} + 5$

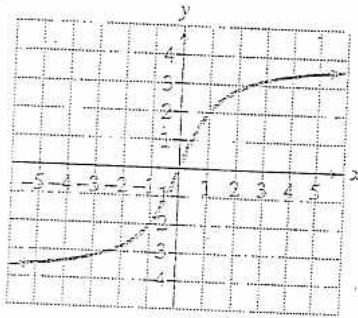
Which graphs in Exercises 88–91 represent functions that have inverse functions?



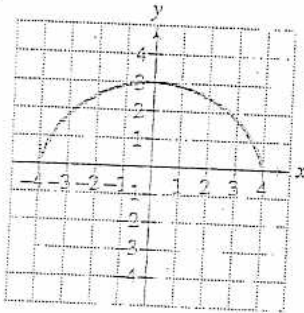
89.



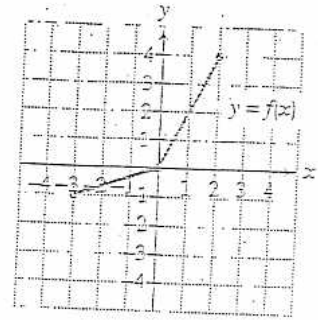
90.



91.



92. Use the graph of f in the figure shown to draw the graph of its inverse function.



In Exercises 93–94, find an equation for $f^{-1}(x)$. Then graph f and f^{-1} in the same rectangular coordinate system.

93. $f(x) = 1 - x^2, x \geq 0$ 94. $f(x) = \sqrt{x} + 1$

In Exercises 95–96, find the distance between each pair of points. If necessary, round answers to two decimal places.

95. $(-2, 3)$ and $(3, -9)$ 96. $(-4, 3)$ and $(-2, 5)$

In Exercises 97–98, find the midpoint of each line segment with the given endpoints.

97. $(2, 6)$ and $(-12, 4)$ 98. $(4, -6)$ and $(-15, 2)$

In Exercises 99–100, write the standard form of the equation of the circle with the given center and radius.

99. Center $(0, 0)$, $r = 3$ 100. Center $(-2, 4)$, $r = 6$

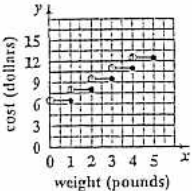
In Exercises 101–103, give the center and radius of each circle and graph its equation. Use the graph to identify the relation's domain and range.

101. $x^2 + y^2 = 1$ 102. $(x + 2)^2 + (y - 3)^2 = 9$

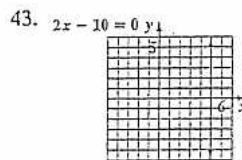
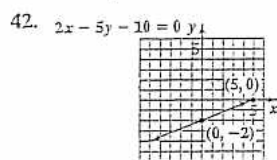
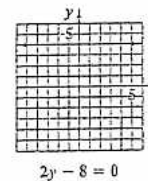
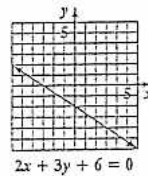
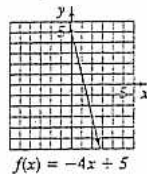
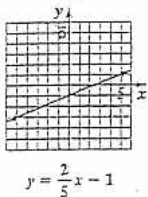
103. $x^2 + y^2 - 4x + 2y - 4 = 0$

Chapter 2 Review Exercises

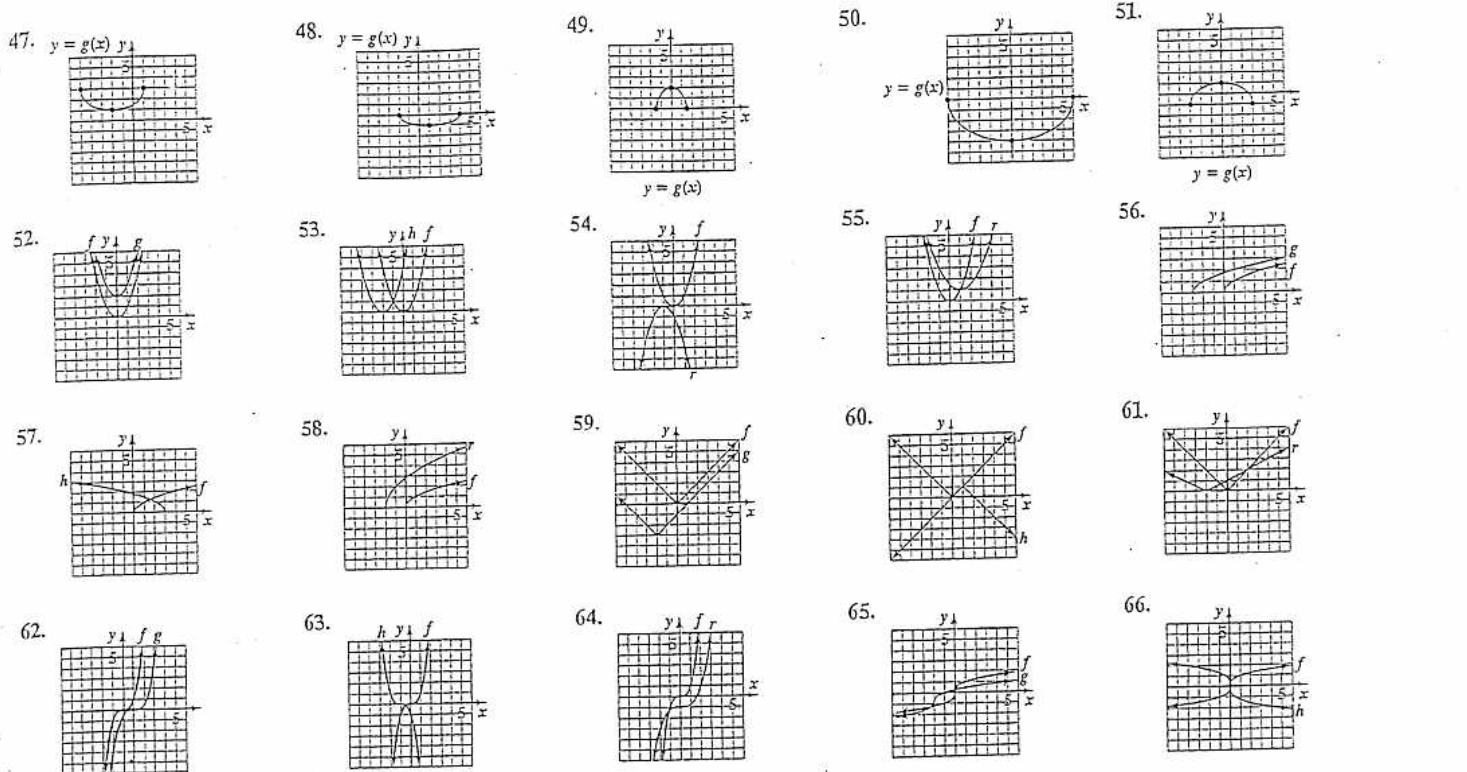
1. Function; Domain: $\{2, 3, 5\}$; Range: $\{7\}$ 2. Function; Domain: $\{1, 2, 13\}$; Range: $\{10, 500, \pi\}$ 3. Not a function; Domain: $\{12, 14\}$; Range: $\{13, 15, 19\}$ 4. y is a function of x . 5. y is a function of x . 6. y is not a function of x . 7. a. $f(4) = -23$ b. $f(x+3) = -7x - 16$
 c. $f(-x) = 5 + 7x$ 8. a. $g(0) = 2$ b. $g(-2) = 24$ c. $g(x-1) = 3x^2 - 11x + 10$ d. $g(-x) = 3x^2 + 5x + 2$
 9. a. $g(13) = 3$ b. $g(0) = 4$ c. $g(-3) = 7$ 10. a. -1 b. 12 c. 3 11. not a function 12. function 13. function
 14. not a function 15. not a function 16. function 17. 8 18. $-4x - 2h + 1$ 19. a. Domain: $[-3, 5)$
 b. Range: $[-5, 0]$ c. x -intercept: -3 d. y -intercept: -2 e. increasing: $(-2, 0)$; or $(3, 5)$ decreasing: $(-3, -2)$ or $(0, 3)$
 f. $f(-2) = -3$ and $f(3) = -5$ 20. a. Domain: $(-\infty, \infty)$ b. Range: $(-\infty, \infty)$ c. x -intercepts: -2 and 3 d. y -intercept: 3
 e. increasing: $(-5, 0)$; decreasing: $(-\infty, -5)$ or $(0, \infty)$ f. $f(-2) = 0$ and $f(6) = -3$ 21. a. Domain: $(-\infty, \infty)$ b. Range: $[-2, 2]$
 c. x -intercept: 0 d. y -intercept: 0 e. increasing: $(-2, 2)$; constant: $(-\infty, -2)$ or $(2, \infty)$ f. $f(-9) = -2$ and $f(14) = 2$
 22. a. 0 ; $f(0) = -2$ b. $-2, 3$; $f(-2) = -3$, $f(3) = -5$ 23. a. 0 ; $f(0) = 3$ b. -5 ; $f(-5) = -6$ 24. odd; symmetric with respect to the origin
 25. even; symmetric with respect to the y -axis 26. odd; symmetric with respect to the origin 27. a. yes; The graph passes the vertical line test. b. Decreasing: $(3, 12)$; The eagle descended. c. Constant: $(0, 3)$ and $(12, 17)$; The eagle's height held steady during the first 3 seconds and the eagle was on the ground for 5 seconds. d. Increasing: $(17, 30)$; The eagle was ascending.

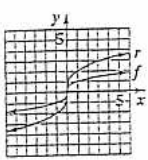
28.  29. $m = -\frac{1}{2}$; falls 30. $m = 1$; rises 31. $m = 0$; horizontal 32. $m =$ undefined; vertical
 33. $y - 2 = -6(x + 3)$; $y = -6x - 16$ 34. using $(1, 6)$, $y - 6 = 2(x - 1)$; $y = 2x + 4$
 35. $y + 7 = -3(x - 4)$; $y = -3x + 5$ 36. $y - 6 = -3(x + 3)$; $y = -3x - 3$ 37. $x + 6y + 18 = 0$

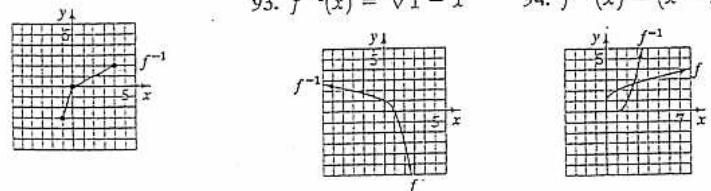
38. Slope: $\frac{2}{5}$; y -intercept: -1 39. Slope: -4 ; y -intercept: 5 40. Slope: $-\frac{2}{3}$; y -intercept: -2 41. Slope: 0 ; y -intercept: 4



44. a. $y - 1.5 = 0.95(x - 1)$, or $y - 3.4 = 0.95(x - 3)$ b. $y = 0.95x + 0.55$
 c. \$10.05 billion 45. a. $m = -44$; The number of new AIDS diagnoses decreased at a rate of 44 each year from 1999 to 2001. b. $m = 909$; The number of new AIDS diagnoses increased at a rate of 909 each year from 2001 to 2003. c. $m = 432.5$; yes;
 Answers will vary. 46. 10

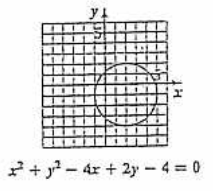
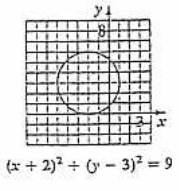
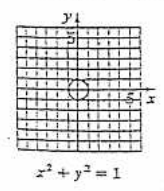


67. 
68. $(-\infty, \infty)$ 69. $(-\infty, 7) \cup (7, \infty)$ 70. $(-\infty, 4]$ 71. $(-\infty, -7) \cup (-7, 3) \cup (3, \infty)$
 72. $[2, 5) \cup (5, \infty)$ 73. $[1, \infty)$ 74. $(f + g)(x) = 4x - 6$; Domain: $(-\infty, \infty)$; $(f - g)(x) = 2x + 4$;
 Domain: $(-\infty, \infty)$; $(fg)(x) = 3x^2 - 16x + 5$; Domain: $(-\infty, \infty)$; $\left(\frac{f}{g}\right)(x) = \frac{3x - 1}{x - 5}$; Domain: $(-\infty, 5) \cup (5, \infty)$
 75. $(f + g)(x) = 2x^2 + x$; Domain: $(-\infty, \infty)$; $(f - g)(x) = x + 2$; Domain: $(-\infty, \infty)$; $(fg)(x) = x^4 + x^3 - x - 1$;
 Domain: $(-\infty, \infty)$; $\left(\frac{f}{g}\right)(x) = \frac{x^2 + x + 1}{x^2 - 1}$; Domain: $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$
76. $(f + g)(x) = \sqrt{x + 7} + \sqrt{x - 2}$; Domain: $[2, \infty)$; $(f - g)(x) = \sqrt{x + 7} - \sqrt{x - 2}$; Domain: $[2, \infty)$; $(fg)(x) = \sqrt{x^2 + 5x - 14}$; Domain: $[2, \infty)$; $\left(\frac{f}{g}\right)(x) = \frac{\sqrt{x + 7}}{\sqrt{x - 2}}$; Domain: $(2, \infty)$
77. a. $(f \circ g)(x) = 16x^2 - 8x + 4$ b. $(g \circ f)(x) = 4x^2 + 11$ c. $(f \circ g)(3) = 124$
78. a. $(f \circ g)(x) = \sqrt{x + 1}$ b. $(g \circ f)(x) = \sqrt{x} + 1$ c. $(f \circ g)(3) = 2$ 79. a. $(f \circ g)(x) = \frac{1 + x}{1 - 2x}$ b. $\left\{x \mid x \neq 0 \text{ and } x \neq \frac{1}{2}\right\}$
80. a. $(f \circ g)(x) = \sqrt{x + 2}$ b. $\{x \mid x \geq -2\}$ 81. $f(x) = x^4, g(x) = x^2 + 2x - 1$ 82. $f(x) = \sqrt[3]{x}, g(x) = 7x + 4$ 83. $f(g(x)) = x - \frac{7}{10}$;
 $g(f(x)) = x - \frac{7}{6}$; f and g are not inverses of each other. 84. $f(g(x)) = x; g(f(x)) = x$; f and g are inverses of each other. 85. $f^{-1}(x) = \frac{x + 3}{4}$
86. $f^{-1}(x) = \sqrt[3]{\frac{x - 1}{8}}$ or $\frac{\sqrt[3]{x - 1}}{2}$ 87. $f^{-1}(x) = \frac{2}{x - 5}$ 88. Inverse function exists. 89. Inverse function does not exist.
90. Inverse function exists. 91. Inverse function does not exist.
92. 93. $f^{-1}(x) = \sqrt{1 - x}$ 94. $f^{-1}(x) = (x - 1)^2, x \geq 1$ 95. 13 96. $2\sqrt{2} \approx 2.83$ 97. $(-5, 5)$



98. $\left(-\frac{11}{2}, -2\right)$ 99. $x^2 + y^2 = 9$
 100. $(x + 2)^2 + (y - 4)^2 = 36$

101. Center: $(0, 0)$; radius: 1
 Domain: $[-1, 1]$; Range: $[-1, 1]$
102. Center: $(-2, 3)$; radius: 3
 Domain: $[-5, 1]$; Range: $[0, 6]$
103. Center: $(2, -1)$; radius: 3
 Domain: $[-1, 5]$; Range: $[-4, 2]$



In Exercises 1–4, use the vertex and intercepts to sketch the graph of each quadratic function. Give the equation for the parabola's axis of symmetry. Use the graph to determine the function's domain and range.

1. $f(x) = -(x + 1)^2 + 4$ 2. $f(x) = (x + 4)^2 - 2$
 3. $f(x) = -x^2 + 2x + 3$ 4. $f(x) = 2x^2 - 4x - 6$

In Exercises 5–6, use the function's equation, and not its graph, to find

- a. the minimum or maximum value and where it occurs.
 b. the function's domain and its range.

5. $f(x) = -x^2 + 14x - 106$

6. $f(x) = 2x^2 + 12x + 703$

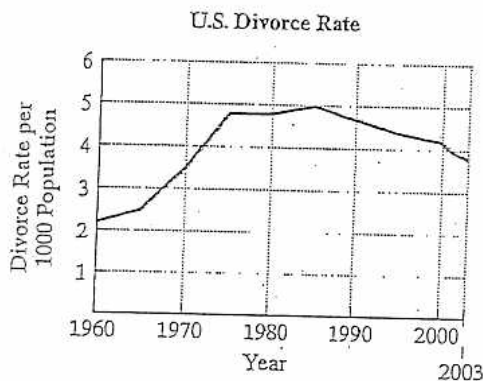
7. The function

$$f(x) = -0.02x^2 + x + 1$$

models the yearly growth of a young redwood tree, $f(x)$, in inches, with x inches of rainfall per year. How many inches of rainfall per year result in maximum tree growth? What is the maximum yearly growth?

8. Suppose that a quadratic function is used to model the data shown in the graph using

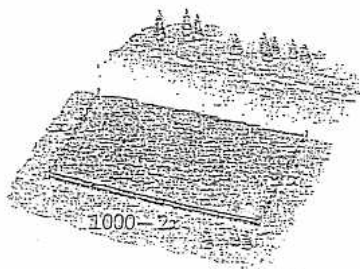
(number of years after 1960, divorce rate per 1000 population).



Source: National Center for Health Statistics

Determine, without obtaining an actual quadratic function that models the data, the approximate coordinates of the vertex for the function's graph. Describe what this means in practical terms.

9. A field bordering a straight stream is to be enclosed. The side bordering the stream is not to be fenced. If 1000 yards of fencing material is to be used, what are the dimensions of the largest rectangular field that can be fenced? What is the maximum area?

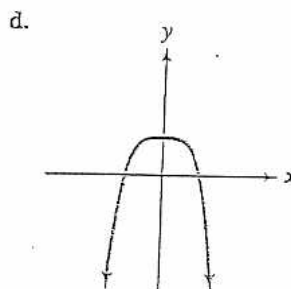
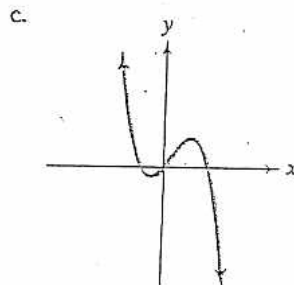
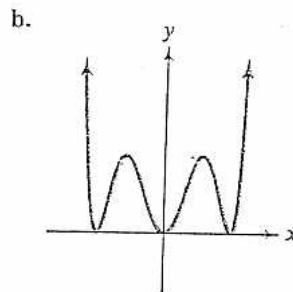
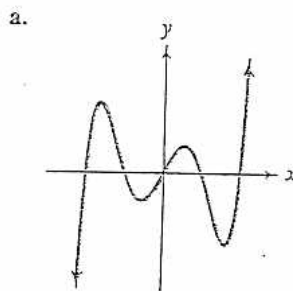


10. Among all pairs of numbers whose difference is 14, find a pair whose product is as small as possible. What is the minimum product?

In Exercises 11–14, use the Leading Coefficient Test to determine the end behavior of the graph of the given polynomial function. Then use this end behavior to match the polynomial function with its graph. [The graphs are labeled (a) through (d).]

11. $f(x) = -x^3 + x^2 + 2x$ 12. $f(x) = x^6 - 6x^4 + 9x^2$

13. $f(x) = x^5 - 5x^3 + 4x$ 14. $f(x) = -x^4 + 1$



15. The polynomial function

$$f(x) = -0.87x^3 + 0.35x^2 + 81.62x + 7684.94$$

models the number of thefts, $f(x)$, in thousands, in the United States x years after 1987. Will this function be useful in modeling the number of thefts over an extended period of time? Explain your answer.

16. A herd of 100 elk is introduced to a small island. The number of elk, $f(x)$, after x years is modeled by the polynomial function

$$f(x) = -x^4 + 21x^2 + 100.$$

Use the Leading Coefficient Test to determine the graph's end behavior to the right. What does this mean about what will eventually happen to the elk population?

In Exercises 17–18, find the zeros for each polynomial function and give the multiplicity of each zero. State whether the graph crosses the x -axis, or touches the x -axis and turns around, at each zero.

17. $f(x) = -2(x - 1)(x + 2)^2(x + 5)^3$

18. $f(x) = x^3 - 5x^2 - 25x + 125$

19. Show that $f(x) = x^3 - 2x - 1$ has a real zero between 1 and 2.

In Exercises 20–25,

a. Use the Leading Coefficient Test to determine the graph's end behavior.

b. Determine whether the graph has y -axis symmetry, origin symmetry, or neither.

c. Graph the function.

20. $f(x) = x^3 - x^2 - 9x + 9$

21. $f(x) = 4x - x^3$

22. $f(x) = 2x^3 + 3x^2 - 8x - 12$

23. $f(x) = -x^4 + 25x^2$

24. $f(x) = -x^4 + 6x^3 - 9x^2$

25. $f(x) = 3x^4 - 15x^3$

In Exercises 26–27, graph each polynomial function.

26. $f(x) = 2x^2(x - 1)^3(x + 2)$

27. $f(x) = -x^3(x + 4)^2(x - 1)$

In Exercises 56–57, use transformations of $f(x) = \frac{1}{x}$ or $f(x) = \frac{1}{x^2}$ to graph each rational function.

56. $g(x) = \frac{1}{(x + 2)^2} - 1$

57. $h(x) = \frac{1}{x - 1} + 3$

In Exercises 58–65, find the vertical asymptotes, if any, the horizontal asymptote, if one exists, and the slant asymptote, if there is one, of the graph of each rational function. Then graph the rational function.

58. $f(x) = \frac{2x}{x^2 - 9}$

59. $g(x) = \frac{2x - 4}{x + 3}$

60. $h(x) = \frac{x^2 - 3x - 4}{x^2 - x - 6}$

61. $r(x) = \frac{x^2 + 4x + 3}{(x + 2)^2}$

62. $y = \frac{x^2}{x + 1}$

63. $y = \frac{x^2 + 2x - 3}{x - 3}$

64. $f(x) = \frac{-2x^3}{x^2 + 1}$

65. $g(x) = \frac{4x^2 - 16x + 16}{2x - 3}$

66. A company is planning to manufacture affordable graphing calculators. The fixed monthly cost will be \$50,000 and it will cost \$25 to produce each calculator.

a. Write the cost function, C , of producing x graphing calculators.

b. Write the average cost function, \bar{C} , of producing x graphing calculators.

c. Find and interpret $\bar{C}(50)$, $\bar{C}(100)$, $\bar{C}(1000)$, and $\bar{C}(100,000)$.

d. What is the horizontal asymptote for the graph of this function and what does it represent?

67. In Palo Alto, California, a government agency ordered computer-related companies to contribute to a monetary pool to clean up underground water supplies. (The companies had stored toxic chemicals in leaking underground containers.) The rational function

$$C(x) = \frac{200x}{100 - x}$$

models the cost, $C(x)$, in tens of thousands of dollars, for removing x percent of the contaminants.

- Find and interpret $C(90) - C(50)$.
- What is the equation for the vertical asymptote? What does this mean in terms of the variables given by the function?

Exercises 68–69 involve rational functions that model the given situations. In each case, find the horizontal asymptote as $x \rightarrow \infty$ and then describe what this means in practical terms.

68. $f(x) = \frac{150x + 120}{0.05x + 1}$; the number of bass, $f(x)$, after x months in a lake that was stocked with 120 bass

69. $P(x) = \frac{72,900}{100x^2 + 729}$; the percentage, $P(x)$, of people in the United States with x years of education who are unemployed

70. The function $p(x) = 1.96x + 3.14$ models the number of non-violent prisoners, $p(x)$, in thousands, in New York State prisons x years after 1980. The function $q(x) = 3.04x + 21.79$ models the total number of prisoners, $q(x)$, in thousands, in New York State prisons x years after 1980.

- Write a function that models the fraction of nonviolent prisoners in New York State prisons x years after 1980.
- What is the equation of the horizontal asymptote associated with the function in part (a)? Describe what this means about the percentage, to the nearest tenth of a percent, of nonviolent prisoners in New York State prisons over time.
- Use your equation in part (b) to explain why, in 1998, New York State implemented a strategy where more nonviolent offenders are granted parole and more violent offenders are denied parole.

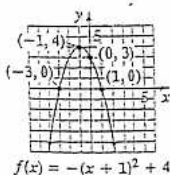
Solve the variation problems in Exercises 78–83.

78. An electric bill varies directly as the amount of electricity used. The bill for 1400 kilowatts of electricity is \$98. What is the bill for 2200 kilowatts of electricity?

79. The distance that a body falls from rest is directly proportional to the square of the time of the fall. If skydivers fall 144 feet in 3 seconds, how far will they fall in 10 seconds?
80. The time it takes to drive a certain distance is inversely proportional to the rate of travel. If it takes 4 hours at 50 miles per hour to drive the distance, how long will it take at 40 miles per hour?
81. The loudness of a stereo speaker, measured in decibels, varies inversely as the square of your distance from the speaker. When you are 8 feet from the speaker, the loudness is 28 decibels. What is the loudness when you are 4 feet from the speaker?
82. The time required to assemble computers varies directly as the number of computers assembled and inversely as the number of workers. If 30 computers can be assembled by 6 workers in 10 hours, how long would it take 5 workers to assemble 40 computers?
83. The volume of a pyramid varies jointly as its height and the area of its base. A pyramid with a height of 15 feet and a base with an area of 35 square feet has a volume of 175 cubic feet. Find the volume of a pyramid with a height of 20 feet and a base with an area of 120 square feet.

Chapter 3 Review Exercises

1.

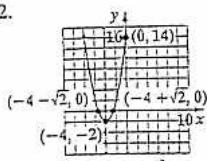


$$f(x) = -(x + 1)^2 + 4$$

axis of symmetry; $x = -1$

Domain: $(-\infty, \infty)$; Range: $(-\infty, 4]$

2.

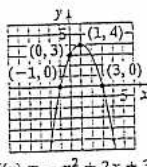


$$f(x) = (x + 4)^2 - 2$$

axis of symmetry: $x = -4$

Domain: $(-\infty, \infty)$; Range: $[-2, \infty)$

3.

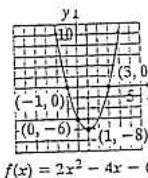


$$f(x) = -x^2 + 2x + 3$$

axis of symmetry: $x = 1$

Domain: $(-\infty, \infty)$; Range: $(-\infty, 4]$

4.



$$f(x) = 2x^2 - 4x - 6$$

axis of symmetry: $x = 1$

Domain: $(-\infty, \infty)$; Range: $[-8, \infty)$

5. a. maximum is -57 at $x = 7$

b. Domain: $(-\infty, \infty)$; Range: $(-\infty, -57]$

6. a. minimum is 685 at $x = -3$

b. Domain: $(-\infty, \infty)$; Range: $[685, \infty)$

7. 25 in. of rainfall per year; 13.5 in. of growth

8. $(25, 5)$; The maximum divorce rate of about 5 per 1000 population was in 1985. 9. 250 yd by 500 yd; 125,000 sq yard 10. -7 and 7 ; -49

11. c 12. b 13. a 14. d 15. No; the graph falls to the right, so eventually there would be a negative number of thefts, which is not possible.

16. The graph falls to the right; eventually the elk population will be extinct.

17. $x = 1$, multiplicity 1, crosses; $x = -2$, multiplicity 2, touches; $x = -5$, multiplicity 3, crosses

18. $x = -5$, multiplicity 1, crosses; $x = 5$, multiplicity 2, touches

19. $f(1)$ is negative and $f(2)$ is positive, so by the Intermediate Value Theorem, f has a real zero between 1 and 2.

20. a. The graph falls to the left and rises to the right.

21. a. The graph rises to the left and falls to the right.

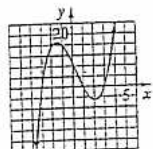
22. a. The graph falls to the left and rises to the right.

b. no symmetry

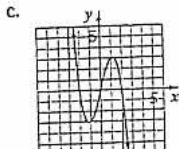
b. origin symmetry

b. no symmetry

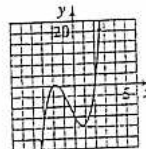
c.



$$f(x) = x^3 - x^2 - 9x + 9$$



$$f(x) = 4x - x^3$$



$$f(x) = 2x^3 + 3x^2 - 8x - 12$$

23. a. The graph falls to the left and to the right.

24. a. The graph falls to the left and to the right.

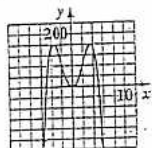
25. a. The graph rises to the left and to the right.

b. y -axis symmetry

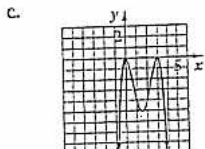
b. no symmetry

b. no symmetry

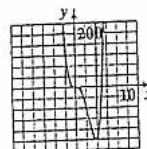
c.



$$f(x) = -x^4 + 25x^2$$

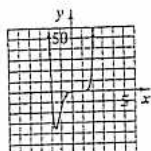


$$f(x) = -x^4 + 6x^3 - 9x^2$$



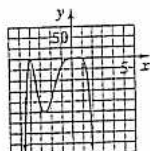
$$f(x) = 3x^4 - 15x^3$$

26.



$$f(x) = 2x^2(x - 1)^3(x + 2)$$

27.



$$f(x) = -x^3(x + 4)^2(x - 1)$$

$$28. 4x^2 - 7x + 5 - \frac{4}{x + 1}$$

$$29. 2x^2 - 4x + 1 - \frac{10}{5x - 3}$$

$$30. 2x^2 + 3x - 1 \quad 31. 3x^3 - 4x^2 + 7$$

$$32. 3x^3 + 6x^2 + 10x + 10 + \frac{20}{x - 2} \quad 33. -5697$$

34. $2, \frac{1}{2}, -3$ 35. $\{4, -2 \pm \sqrt{5}\}$ 36. $\pm 1, \pm 5$ 37. $\pm 1, \pm 2, \pm 4, \pm 8, \pm \frac{8}{3}, \pm \frac{4}{3}, \pm \frac{2}{3}, \pm \frac{1}{3}$ 38. 2 or 0 positive solutions; no negative solutions

39. 3 or 1 positive real roots; 2 or 0 negative solutions 40. No sign variations exist for either $f(x)$ or $f(-x)$, so no real roots exist.

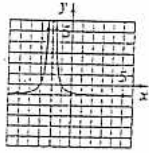
41. a. $\pm 1, \pm 2, \pm 4$ b. 1 positive real zero; 2 or no negative real zeros c. 1 is a zero d. $\{1, -2\}$

42. a. $\pm 1, \pm \frac{1}{2}, \pm \frac{1}{3}, \pm \frac{1}{6}$ b. 2 or 0 positive real zeros; 1 negative real zero c. -1 is a zero d. $\{-1, \frac{1}{3}, \frac{1}{2}\}$

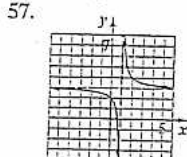
43. a. $\pm 1, \pm 3, \pm 5, \pm 15, \pm \frac{1}{2}, \pm \frac{1}{4}, \pm \frac{1}{8}, \pm \frac{3}{2}, \pm \frac{3}{4}, \pm \frac{3}{8}, \pm \frac{5}{2}, \pm \frac{5}{4}, \pm \frac{5}{8}, \pm \frac{15}{2}, \pm \frac{15}{4}, \pm \frac{15}{8}$ b. 3 or 1 positive real solutions; no negative real solutions

c. $\frac{1}{2}$ is a zero d. $\{\frac{1}{2}, \frac{3}{2}, \frac{5}{2}\}$

44. a. $\pm 1, \pm \frac{1}{2}$ b. 2 or 0 positive real solutions; 1 negative solution c. $\frac{1}{2}$ is a zero d. $\left\{ \frac{1}{2}, \frac{-5 - \sqrt{29}}{2}, \frac{-5 + \sqrt{29}}{2} \right\}$
45. a. $\pm 1, \pm 2, \pm 3, \pm 6$ b. 2 or zero positive real solutions; 2 or zero negative real solutions c. -2 is a zero d. $\{-2, -1, 1, 3\}$
46. a. $\pm 1, \pm 2, \pm \frac{1}{2}, \pm \frac{1}{4}$ b. 1 positive real root; 1 negative real root c. $\frac{1}{2}$ is a zero d. $\left\{ \frac{1}{2}, -\frac{1}{2}, i\sqrt{2}, -i\sqrt{2} \right\}$
47. a. $\pm 1, \pm 2, \pm 4, \pm \frac{1}{2}$ b. 2 or no positive zeros; 2 or no negative zeros c. 2 is a zero d. $\left\{ 2, -2, \frac{1}{2}, -1 \right\}$
48. $f(x) = x^3 - 6x^2 + 21x - 26$ 49. $f(x) = 2x^4 + 12x^3 + 20x^2 + 12x + 18$ 50. $-2, \frac{1}{2}, \pm i; f(x) = (x - i)(x + i)(x + 2)(2x - 1)$
51. $-1, 4; g(x) = (x + 1)^2(x - 4)^2$ 52. 4 real zeros, one with multiplicity two 53. 3 real zeros; 2 nonreal complex zeros
54. 2 real zeros, one with multiplicity two; 2 nonreal complex zeros 55. 1 real zero; 4 nonreal complex zeros

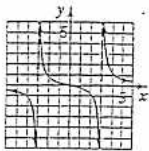


$$g(x) = \frac{1}{(x+2)^2} - 1$$



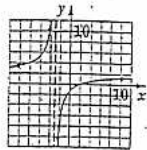
$$h(x) = \frac{1}{x-1} + 3$$

58. Vertical asymptote: $x = 3$ and $x = -3$
horizontal asymptote: $y = 0$



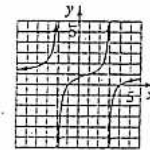
$$f(x) = \frac{2x}{x^2 - 9}$$

59. Vertical asymptote: $x = -3$
horizontal asymptote: $y = 2$



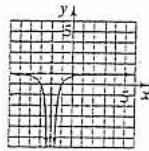
$$f(x) = \frac{2x - 4}{x + 3}$$

60. Vertical asymptotes: $x = 3, -2$
horizontal asymptote: $y = 1$



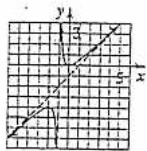
$$h(x) = \frac{x^2 - 3x - 4}{x^2 - x - 6}$$

61. Vertical asymptote: $x = -2$
horizontal asymptote: $y = 1$



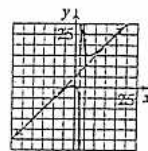
$$r(x) = \frac{x^2 + 4x + 3}{(x + 2)^2}$$

62. Vertical asymptote: $x = -1$
no horizontal asymptote
slant asymptote: $y = x - 1$



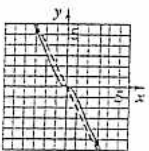
$$y = \frac{x^2}{x + 1}$$

63. Vertical asymptote: $x = 3$
no horizontal asymptote
slant asymptote: $y = x + 5$



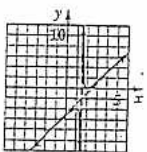
$$y = \frac{x^2 + 2x - 3}{x - 3}$$

64. No vertical asymptote
no horizontal asymptote
slant asymptote: $y = -2x$



$$f(x) = \frac{-2x^3}{x^2 + 1}$$

65. Vertical asymptote: $x = \frac{3}{2}$
no horizontal asymptote
slant asymptote: $y = 2x - 5$



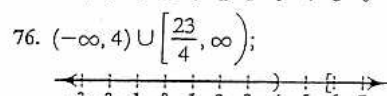
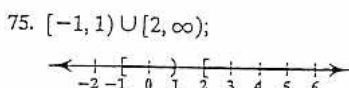
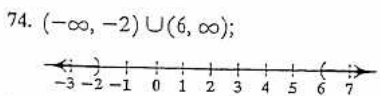
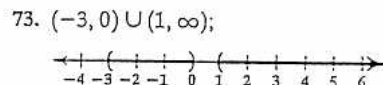
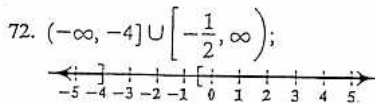
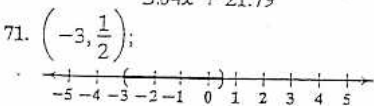
$$g(x) = \frac{4x^2 - 16x + 16}{2x - 3}$$

66. a. $C(x) = 25x + 50,000$
b. $\bar{C}(x) = \frac{25x + 50,000}{x}$

c. $\bar{C}(50) = 1025$, when 50 calculators are manufactured, it costs \$1025 to manufacture each;
 $\bar{C}(100) = 525$, when 100 calculators are manufactured, it costs \$525 to manufacture each;
 $\bar{C}(1000) = 75$, when 1000 calculators are manufactured, it costs \$75 to manufacture each;
 $\bar{C}(100,000) = 25.5$, when 100,000 calculators are manufactured, it costs \$25.50 to manufacture each.

d. $y = 25$; costs will approach \$25.

67. a. 1600; The difference in cost of removing 90% versus 50% of the contaminants is 16 million dollars.
b. $x = 100$; No amount of money can remove 100% of the contaminants, since $C(x)$ increases without bound as x approaches 100.
68. $y = 3000$; The number of fish in the pond approaches 3000.
69. $y = 0$; As the number of years of education increases the percentage rate of unemployment approaches zero.
70. a. $f(x) = \frac{1.96x + 3.14}{3.04x + 21.79}$ b. $y = 0.6$; As the years increase, the fraction of nonviolent prisoners approaches 0.6. c. Answers may vary.



77. from 1 to 2 sec 78. \$154 79. 1600 ft 80. 5 hr 81. 112 decibels 82. 16 hr 83. 800 ft³

Use the graph shown at the bottom of the previous page to answer each of the following questions.

- What was the temperature of the coffee when it was first taken out of the microwave?
- What is a reasonable estimate of the temperature of the coffee after 20 minutes? Use your calculator to verify this estimate.
- What is the limit of the temperature to which the coffee will cool? What does this tell you about the temperature of the room?

In Exercises 13–15, write each equation in its equivalent exponential form.

13. $\frac{1}{2} = \log_{49} 7$ 14. $3 = \log_4 x$ 15. $\log_3 81 = y$

In Exercises 16–18, write each equation in its equivalent logarithmic form.

16. $6^3 = 216$ 17. $b^4 = 625$ 18. $13^y = 874$

In Exercises 19–29, evaluate each expression without using a calculator. If evaluation is not possible, state the reason.

19. $\log_4 64$ 20. $\log_5 \frac{1}{25}$ 21. $\log_3(-9)$

22. $\log_{16} 4$ 23. $\log_{17} 17$ 24. $\log_3 3^8$

25. $\ln e^5$ 26. $\log_3 \frac{1}{\sqrt{3}}$ 27. $\ln \frac{1}{e^2}$

28. $\log \frac{1}{1000}$ 29. $\log_3(\log_3 8)$

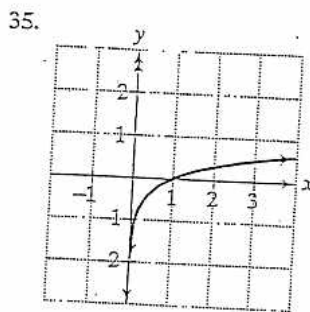
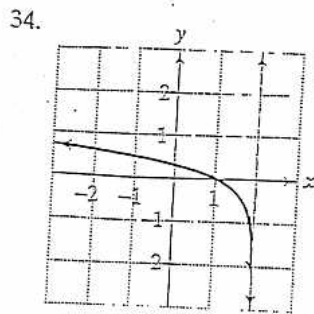
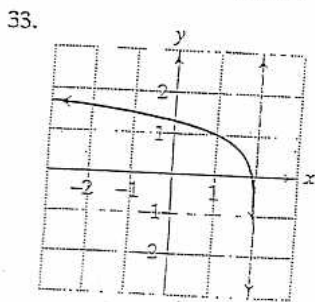
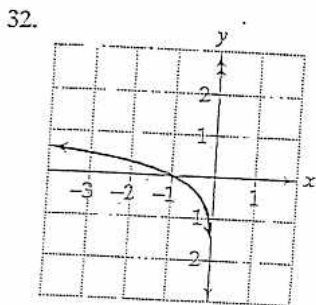
30. Graph $f(x) = 2^x$ and $g(x) = \log_2 x$ in the same rectangular coordinate system.

31. Graph $f(x) = \left(\frac{1}{5}\right)^x$ and $g(x) = \log_{\frac{1}{5}} x$ in the same rectangular coordinate system.

In Exercises 32–35, the graph of a logarithmic function is given. Select the function for each graph from the following options:

$f(x) = \log x$, $g(x) = \log(-x)$,

$h(x) = \log(2 - x)$, $r(x) = 1 + \log(2 - x)$.



In Exercises 36–38, begin by graphing $f(x) = \log_2 x$. Then use transformations of this graph to graph the given function. What is the graph's x-intercept? What is the vertical asymptote? Use the graphs to determine each function's domain and range.

36. $g(x) = \log_2(x - 2)$ 37. $h(x) = -1 + \log_2 x$

38. $r(x) = \log_2(-x)$

In Exercises 39–40, graph f and g in the same rectangular coordinate system. Use transformations of the graph of f to obtain the graph of g . Graph and give equations of all asymptotes. Use the graphs to determine each function's domain and range.

39. $f(x) = \log x$ and $g(x) = -\log(x + 3)$

40. $f(x) = \ln x$ and $g(x) = -\ln(2x)$

In Exercises 41–43, find the domain of each logarithmic function.

41. $f(x) = \log_8(x + 5)$ 42. $f(x) = \log(3 - x)$

43. $f(x) = \ln(x - 1)^2$

In Exercises 44–46, use inverse properties of logarithms to simplify each expression.

44. $\ln e^{6x}$ 45. $e^{\ln \sqrt{x}}$ 46. $10^{\log 4x^2}$

47. On the Richter scale, the magnitude, R , of an earthquake of intensity I is given by $R = \log \frac{I}{I_0}$, where I_0 is the intensity of a barely felt zero-level earthquake. If the intensity of an earthquake is $1000I_0$, what is its magnitude on the Richter scale?

48. Students in a psychology class took a final examination. As part of an experiment to see how much of the course content they remembered over time, they took equivalent forms of the exam in monthly intervals thereafter. The average score, $f(t)$, for the group after t months is modeled by the function $f(t) = 76 - 18 \log(t + 1)$, where $0 \leq t \leq 12$.

- What was the average score when the exam was first given?

- b. What was the average score after 2 months? 4 months? 6 months? 8 months? one year?
- c. Use the results from parts (a) and (b) to graph f . Describe what the shape of the graph indicates in terms of the material retained by the students.

49. The formula

$$t = \frac{1}{c} \ln \left(\frac{A}{A - N} \right)$$

describes the time, t , in weeks, that it takes to achieve mastery of a portion of a task. In the formula, A represents maximum learning possible, N is the portion of the learning that is to be achieved, and c is a constant used to measure an individual's learning style. A 50-year-old man decides to start running as a way to maintain good health. He feels that the maximum rate he could ever hope to achieve is 12 miles per hour. How many weeks will it take before the man can run 5 miles per hour if $c = 0.06$ for this person?

In Exercises 50–53, use properties of logarithms to expand each logarithmic expression as much as possible. Where possible, evaluate logarithmic expressions without using a calculator.

50. $\log_6(36x^3)$

51. $\log_4 \left(\frac{\sqrt{x}}{64} \right)$

52. $\log_2 \left(\frac{xy^2}{64} \right)$

53. $\ln \sqrt[3]{\frac{x}{e}}$

In Exercises 54–57, use properties of logarithms to condense each logarithmic expression. Write the expression as a single logarithm whose coefficient is 1.

54. $\log_b 7 + \log_b 3$

55. $\log 3 - 3 \log x$

56. $3 \ln x + 4 \ln y$

57. $\frac{1}{2} \ln x - \ln y$

In Exercises 58–59, use common logarithms or natural logarithms and a calculator to evaluate to four decimal places.

58. $\log_6 72,348$

59. $\log_4 0.863$

In Exercises 60–63, determine whether each equation is true or false. Where possible, show work to support your conclusion. If the statement is false, make the necessary change(s) to produce a true statement.

60. $(\ln x)(\ln 1) = 0$

61. $\log(x + 9) - \log(x + 1) = \frac{\log(x + 9)}{\log(x + 1)}$

62. $(\log_2 x)^4 = 4 \log_2 x$

63. $\ln e^x = x \ln e$

In Exercises 64–72, solve each exponential equation. Where necessary, express the solution set in terms of natural logarithms and use a calculator to obtain a decimal approximation, correct to two decimal places, for the solution.

64. $2^{4x-2} = 64$

65. $125^x = 25$

66. $9^{x+2} = 27^{-x}$

67. $8^x = 12,143$

68. $9e^{5x} = 1269$

69. $e^{12-5x} - 7 = 123$

70. $5^{4x+2} = 37,500$

71. $3^{x+4} = 7^{2x-1}$

72. $e^{2x} - e^x - 6 = 0$

In Exercises 73–78, solve each logarithmic equation.

73. $\log_4(3x - 5) = 3$

74. $3 + 4 \ln(2x) = 15$

75. $\log_2(x + 3) + \log_2(x - 3) = 4$

76. $\log_3(x - 1) - \log_3(x + 2) = 2$

77. $\ln(x + 4) - \ln(x + 1) = \ln x$

78. $\log_4(2x + 1) = \log_4(x - 3) + \log_4(x + 5)$

79. The function $P(x) = 14.7e^{-0.21x}$ models the average atmospheric pressure, $P(x)$, in pounds per square inch, at an altitude of x miles above sea level. The atmospheric pressure at the peak of Mt. Everest, the world's highest mountain, is 4.6 pounds per square inch. How many miles above sea level, to the nearest tenth of a mile, is the peak of Mt. Everest?

80. The amount of carbon dioxide in the atmosphere, measured in parts per million, has been increasing as a result of the burning of oil and coal. The buildup of gases and particles traps heat and raises the planet's temperature, a phenomenon called the greenhouse effect. Carbon dioxide accounts for about half of the warming. The function $f(t) = 364(1.005)^t$ projects carbon dioxide concentration, $f(t)$, in parts per million, t years after 2000. Using the projections given by the function, when will the carbon dioxide concentration be double the preindustrial level of 280 parts per million?

81. The function $W(x) = 0.37 \ln x + 0.05$ models the average walking speed, $W(x)$, in feet per second, of residents in a city whose population is x thousand. Visitors to New York City frequently feel they are moving too slowly to keep pace with New Yorkers' average walking speed of 3.38 feet per second. What is the population of New York City? Round to the nearest thousand.

82. Use the formula for compound interest with n compoundings per year to solve this problem. How long, to the nearest tenth of a year, will it take \$12,500 to grow to \$20,000 at 6.5% annual interest compounded quarterly?

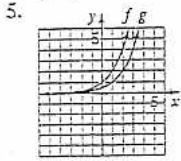
Use the formula for continuous compounding to solve Exercises 83–84.

83. How long, to the nearest tenth of a year, will it take \$50,000 to triple in value at 7.5% annual interest compounded continuously?

84. What interest rate, to the nearest percent, is required for an investment subject to continuous compounding to triple in 5 years?

Chapter 4 Review Exercises

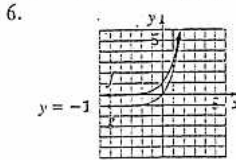
1. $g(x) = 4^{-x}$ 2. $h(x) = -4^{-x}$ 3. $r(x) = -4^{-x} + 3$ 4. $f(x) = 4^x$



$$f(x) = 2^x$$

$$g(x) = 2^x - 1$$

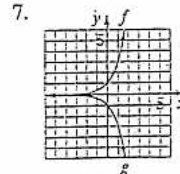
asymptote of f : $y = 0$;
 asymptote of g : $y = 0$;
 Domain of f = Domain of $g = (-\infty, \infty)$;
 Range of f = Range of $g = (0, \infty)$



$$f(x) = 3^x$$

$$g(x) = 3^x - 1$$

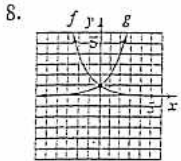
asymptote of f : $y = 0$;
 asymptote of g : $y = -1$;
 Domain of f = Domain of $g = (-\infty, \infty)$;
 Range of f = $(0, \infty)$; Range of $g = (-1, \infty)$



$$f(x) = 3^x$$

$$g(x) = -3^x$$

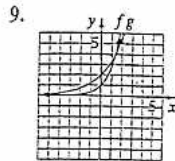
asymptote of f : $y = 0$;
 asymptote of g : $y = 0$;
 Domain of f = Domain of $g = (-\infty, \infty)$;
 Range of $f = (0, \infty)$; Range of $g = (-\infty, 0)$



$$f(x) = \left(\frac{1}{2}\right)^x$$

$$g(x) = \left(\frac{1}{2}\right)^{-x}$$

asymptote of f : $y = 0$;
 asymptote of g : $y = 0$;
 Domain of f = Domain of $g = (-\infty, \infty)$;
 Range of f = Range of $g = (0, \infty)$

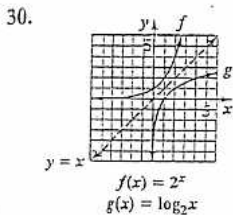


$$f(x) = e^x$$

$$g(x) = 2e^{x/2}$$

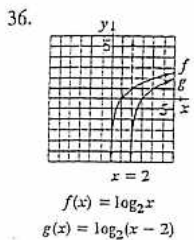
asymptote of f : $y = 0$;
 asymptote of g : $y = 0$;
 Domain of f = Domain of $g = (-\infty, \infty)$;
 Range of f = Range of $g = (0, \infty)$

10. 5.5% compounded semiannually 11. 7% compounded monthly 12. a. 200° b. $120^\circ, 119^\circ$ c. 70° ; The temperature in the room is 70° .
 13. $49^{1/2} = 7$ 14. $4^3 = x$ 15. $3^y = 81$ 16. $\log_6 216 = 3$ 17. $\log_6 625 = 4$ 18. $\log_{13} 874 = y$ 19. 3 20. -2 21. undefined;
 $\log_5 x$ is defined only for $x > 0$. 22. $\frac{1}{2}$ 23. 1 24. 8 25. 5 26. $-\frac{1}{2}$ 27. -2 28. -3 29. 0



$$f(x) = 2^x$$

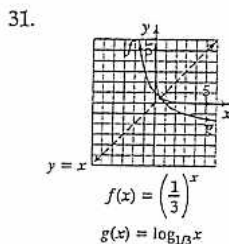
$$g(x) = \log_2 x$$



$$f(x) = \log_2 x$$

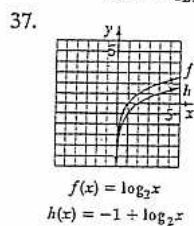
$$g(x) = \log_2(x-2)$$

x -intercept: $(3, 0)$
 vertical asymptote: $x = 2$
 Domain: $(2, \infty)$; Range: $(-\infty, \infty)$



$$f(x) = \left(\frac{1}{3}\right)^x$$

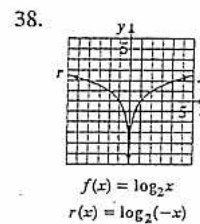
$$g(x) = \log_{1/3} x$$



$$f(x) = \log_2 x$$

$$h(x) = -1 + \log_2 x$$

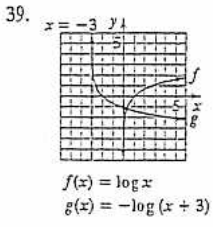
x -intercept: $(2, 0)$
 vertical asymptote: $x = 0$
 Domain: $(0, \infty)$; Range: $(-\infty, \infty)$



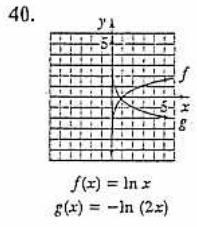
$$f(x) = \log_2 x$$

$$r(x) = \log_2(-x)$$

x -intercept: $(-1, 0)$
 vertical asymptote: $x = 0$
 Domain: $(-\infty, 0)$; Range: $(-\infty, \infty)$

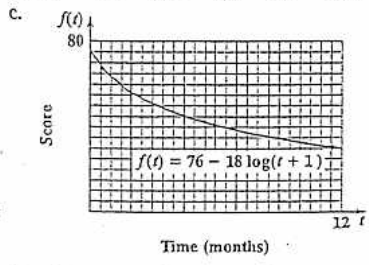


asymptote of f : $x = 0$;
asymptote of g : $x = -3$;
Domain of $f = (0, \infty)$; Domain of $g = (-3, \infty)$;
Range of $f = \text{Range of } g = (-\infty, \infty)$



asymptote of f : $x = 0$;
asymptote of g : $x = 0$;
Domain of $f = \text{Domain of } g = (0, \infty)$;
Range of $f = \text{Range of } g = (-\infty, \infty)$

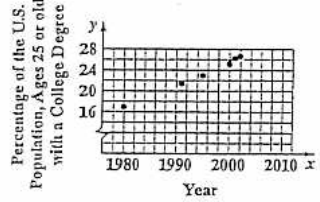
41. $(-5, \infty)$ 42. $(-\infty, 3)$ 43. $(-\infty, 1) \cup (1, \infty)$ 44. $6x$ 45. \sqrt{x} 46. $4x^2$ 47. 3.0
48. a. 76 b. $\approx 67, \approx 63, \approx 61, \approx 59, \approx 56$ 49. about 9 weeks 50. $2 + 3 \log_6 x$ 51. $\frac{1}{2} \log_4 x - 3$



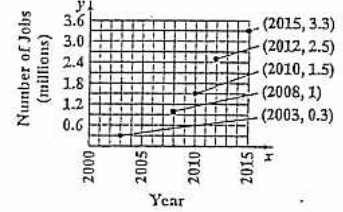
52. $\log_2 x + 2 \log_2 y - 6$ 53. $\frac{1}{3} \ln x - \frac{1}{3}$ 54. $\log_b 21$ 55. $\log \frac{3}{x}$
56. $\ln(x^3 y^4)$ 57. $\ln \frac{\sqrt{x}}{y}$ 58. 6.2448 59. -0.1063
60. true 61. false; $\log(x+9) - \log(x+1) = \log\left(\frac{x+9}{x+1}\right)$
62. false; $4 \log_2 x = \log_2 x^4$ 63. true 64. $\{2\}$ 65. $\left\{\frac{2}{3}\right\}$

66. $\left\{-\frac{4}{5}\right\}$ 67. $\left\{\frac{\ln 12,143}{\ln 8}\right\}; \approx 4.52$ 68. $\left\{\frac{1}{5} \ln 141\right\}; \approx 0.99$ 69. $\left\{\frac{12 - \ln 130}{5}\right\}; \approx 1.43$ 70. $\left\{\frac{\ln 37,500 - 2 \ln 5}{4 \ln 5}\right\}; \approx 1.14$
71. $\left\{\frac{\ln 7 + 4 \ln 3}{2 \ln 7 - \ln 3}\right\}; \approx 2.27$ 72. $\{\ln 3\}; \approx 1.10$ 73. $\{23\}$ 74. $\left\{\frac{e^2}{2}\right\}; \approx 10.04$ 75. $\{5\}$ 76. \emptyset 77. $\{2\}$ 78. $\{4\}$
79. 5.5 mi 80. approximately 2086 81. approximately 8103 thousand or 8,103,000 82. 7.3 yr 83. 14.6 yr 84. about 22%

85. a. 0.045 b. 55.1 million c. 2012
88. a.



86. 325 days 87. a. 200 people b. about 45,411 people c. 500,000 people
89. a.



90. $y = 73e^{(\ln 2.6)x}; y = 73e^{0.956x}$
91. $y = 6.5e^{(\ln 0.45)x}; y = 6.5e^{-0.844x}$

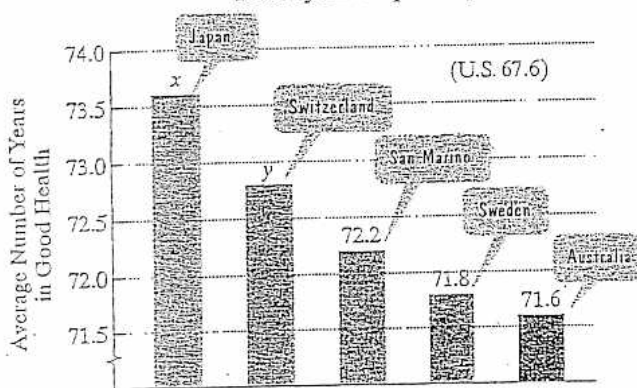
- b. exponential function b. exponential function

In Exercises 1–5, solve by the method of your choice. Identify systems with no solution and systems with infinitely many solutions, using set notation to express their solution sets.

- $y = 4x + 1$
 $3x + 2y = 13$
- $x + 4y = 14$
 $2x - y = 1$
- $5x + 3y = 1$
 $3x + 4y = -6$
- $2y - 6z = 7$
 $3x - y = 9$
- $4x - 8y = 16$
 $3x - 6y = 12$
- A company is planning to manufacture computer desks. The fixed cost will be \$60,000 and it will cost \$200 to produce each desk. Each desk will be sold for \$450.

 - Write the cost function, C , of producing x desks.
 - Write the revenue function, R , from the sale of x desks.
 - Determine the break-even point. Describe what this means.
- The bar graph shows the five countries with the longest healthy life expectancy at birth. Combined, people in Japan and Switzerland can expect to spend 146.4 years in good health. The difference between healthy life expectancy between these two countries is 0.8 years. Find the healthy life expectancy at birth in Japan and Switzerland.

Countries with the Longest Healthy Life Expectancy



Source: World Health Organization

- The perimeter of a rectangular table top is 34 feet. The difference between 4 times the length and 3 times the width is 33 feet. Find the dimensions.
- A travel agent offers two package vacation plans. The first plan costs \$360 and includes 3 days at a hotel and a rental car for 2 days. The second plan costs \$500 and includes 4 days at a hotel and a rental car for 3 days. The daily charge for the hotel is the same under each plan, as is the daily charge for the car. Find the cost per day for the hotel and for the car.
- The calorie-nutrient information for an apple and an avocado is given in the table. How many of each should be eaten to get exactly 1000 calories and 100 grams of carbohydrates?

	One Apple	One Avocado
Calories	100	350
Carbohydrates (grams)	24	14

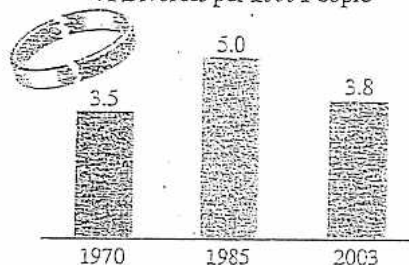
Solve each system in Exercises 11–12.

- $2x - y + z = 1$
 $3x - 3y + 4z = 5$
 $4x - 2y + 3z = 4$
- $x + 2y - z = 5$
 $2x - y + 3z = 0$
 $2y + z = 1$

- Find the quadratic function $y = ax^2 + bx + c$ whose

- The bar graph shows that the U.S. divorce rate increased from 1970 to 1985 and then decreased from 1985 to 2003.

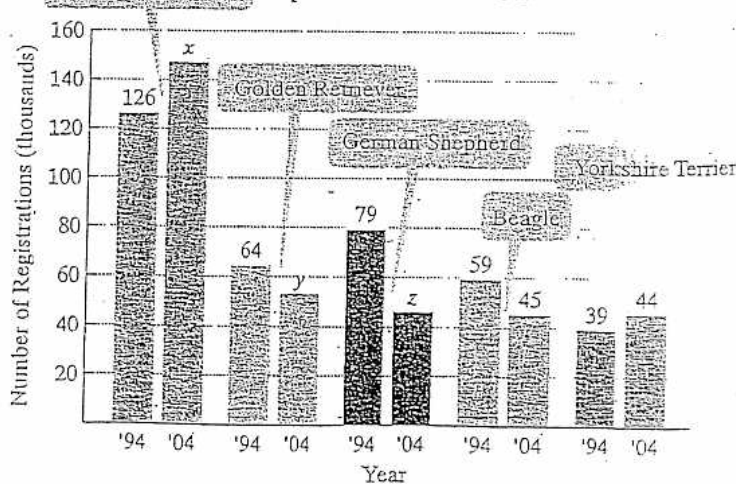
U.S. Divorce Rates: Number of Divorces per 1000 People



Source: U.S. Census Bureau

- Write the data for 1970, 1985, and 2003 as ordered pairs (x, y) , where x is the number of years after 1970 and y is that year's divorce rate.
 - The three data points in part (a) can be modeled by the quadratic function $y = ax^2 + bx + c$. Write a system of linear equations in three variables that can be used to find values for a , b , and c . It is not necessary to solve the system.
- The bar graph shows the top five purebred dogs in the United States in 2004 compared with how many there were a decade earlier.

Popular Pooches in the U.S.



Source: American Kennel Club

In 2004, for the 15th year in a row, the Labrador retriever was the most popular purebred dog. The number of Labs exceeded the number of Golden retrievers and German shepherds combined by 48 thousand. Furthermore, the number of Labs was 9 thousand more than three times the number of German shepherds. If there were 246 thousand registrations for Labrador retrievers, Golden retrievers, and German shepherds, how many registrations, in thousands, were there for each of these breeds?

In Exercises 39–45, graph each inequality.

39. $3x - 4y > 12$

40. $y \leq -\frac{1}{2}x + 2$

41. $x < -2$

42. $y \geq 3$

43. $x^2 + y^2 > 4$

44. $y \leq x^2 - 1$

45. $y \leq 2^x$

In Exercises 46–55, graph the solution set of each system of inequalities or indicate that the system has no solution.

46. $3x + 2y \geq 6$

47. $2x - y \geq 4$

$2x + y \geq 6$

$x + 2y < 2$

48. $y < x$

49. $x + y \leq 6$

$y \leq 2$

$y \geq 2x - 3$

50. $0 \leq x \leq 3$

51. $2x + y < 4$

$y > 2$

$2x + y > 6$

52. $x^2 + y^2 \leq 16$

53. $x^2 + y^2 \leq 9$

$x + y < 2$

$y < -3x + 1$

54. $y > x^2$

55. $y \geq 0$

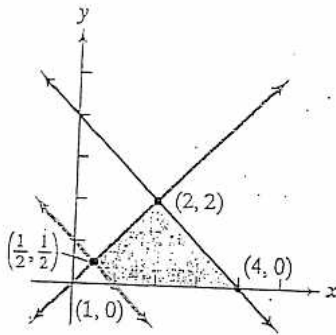
$x + y < 6$

$3x + 2y \geq 4$

$y < x + 6$

$x - y \leq 3$

56. Find the value of the objective function $z = 2x + 3y$ at each corner of the graphed region shown. What is the maximum value of the objective function? What is the minimum value of the objective function?



In Exercises 57–59, graph the region determined by the constraints. Then find the maximum value of the given objective function, subject to the constraints.

57. Objective Function
Constraints

$z = 2x + 3y$

$x \geq 0, y \geq 0$

$x + y \leq 8$

$3x + 2y \geq 6$

58. Objective Function
Constraints

$z = x + 4y$

$0 \leq x \leq 5, 0 \leq y \leq 7$

$x + y \geq 3$

59. Objective Function
Constraints

$z = 5x + 6y$

$x \geq 0, y \geq 0$

$y \leq x$

$2x + y \leq 12$

60. A paper manufacturing company converts wood pulp to writing paper and newsprint. The profit on a unit of writing paper is \$500 and the profit on a unit of newsprint is \$350.

a. Let x represent the number of units of writing paper produced daily. Let y represent the number of units of newsprint produced daily. Write the objective function that models total daily profit.

b. The manufacturer is bound by the following constraints:

- Equipment in the factory allows for making at most 200 units of paper (writing paper and newsprint) in a day.
- Regular customers require at least 10 units of writing paper and at least 80 units of newsprint daily.

Write a system of inequalities that models these constraints.

c. Graph the inequalities in part (b). Use only the first quadrant, because x and y must both be positive. (Suggestion: Let each unit along the x - and y -axes represent 20.)

d. Evaluate the objective function at each of the three vertices of the graphed region.

e. Complete the missing portions of this statement: The company will make the greatest profit by producing _____ units of writing paper and _____ units of newsprint each day. The maximum daily profit is \$_____.

61. A manufacturer of lightweight tents makes two models whose specifications are given in the following table:

	Cutting Time per Tent	Assembly Time per Tent
Model A	0.9 hour	0.8 hour
Model B	1.8 hours	1.2 hours

On a monthly basis, the manufacturer has no more than 864 hours of labor available in the cutting department and at most 672 hours in the assembly division. The profits come to \$25 per tent for model A and \$40 per tent for model B. How many of each should be manufactured monthly to maximize the profit?

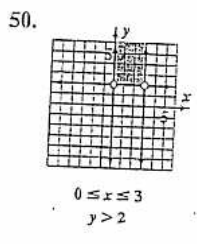
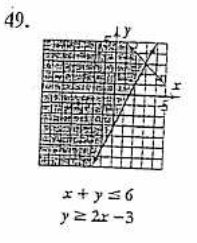
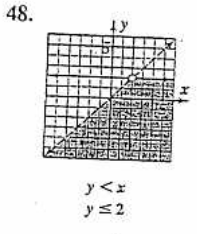
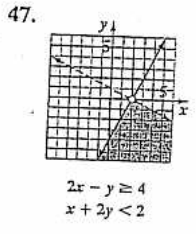
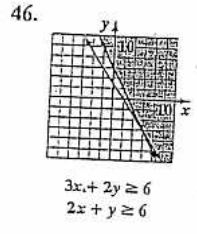
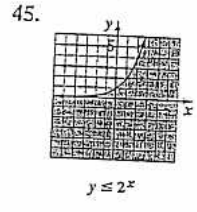
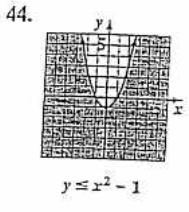
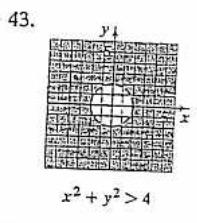
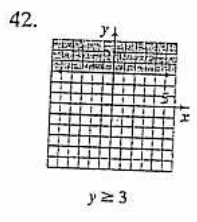
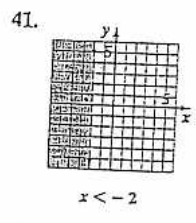
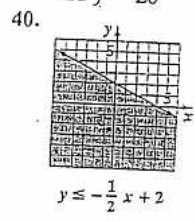
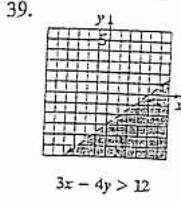
Chapter 5 Review Exercises

1. $\{(1, 5)\}$ 2. $\{(2, 3)\}$ 3. $\{(2, -3)\}$ 4. \emptyset 5. $\{(x, y) | 3x - 6y = 12\}$ 6. a. $C(x) = 60,000 + 200x$ b. $R(x) = 450x$
 c. (240, 108,000); This means the company will break even if it produces and sells 240 desks. 7. Japan: 73.6 yr; Switzerland: 72.8 yr
 8. 12 ft by 5 ft 9. \$80 per day for the room, \$60 per day for the car 10. 3 apples and 2 avocados 11. $\{(0, 1, 2)\}$ 12. $\{(2, 1, -1)\}$

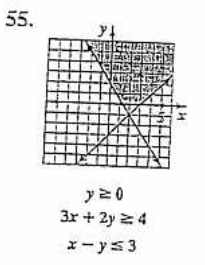
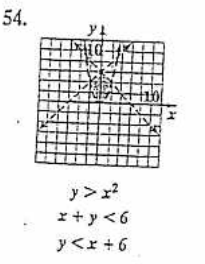
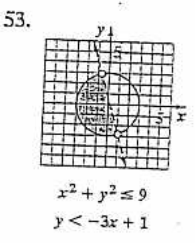
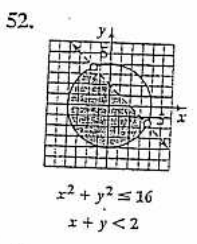
13. $y = 3x^2 - 4x + 5$ 14. a. $(0, 3.5), (15, 5.0), (33, 3.8)$ b. $c = 3.5; 225a + 15b + c = 5.0; 1089a + 33b + c = 3.8$

15. Labrador retrievers: 147; Golden retrievers: 53; German shepherds: 46 16. $\frac{3}{5(x-3)} + \frac{2}{5(x+2)}$ 17. $\frac{6}{x-4} + \frac{5}{x+3}$
 18. $\frac{2}{x} + \frac{3}{x+2} - \frac{1}{x-1}$ 19. $\frac{2}{x-2} + \frac{5}{(x-2)^2}$ 20. $\frac{4}{x-1} + \frac{4}{x-2} - \frac{2}{(x-2)^2}$ 21. $\frac{6}{5(x-2)} + \frac{-6x+3}{5(x^2+1)}$ 22. $\frac{5}{x-3} + \frac{2x-1}{x^2+4}$
 23. $\frac{x}{x^2+4} - \frac{4x}{(x^2+4)^2}$ 24. $\frac{4x+1}{x^2+x+1} + \frac{2x-2}{(x^2+x+1)^2}$ 25. $\{(4, 3), (1, 0)\}$ 26. $\{(0, 1), (-3, 4)\}$ 27. $\{(1, -1), (-1, 1)\}$

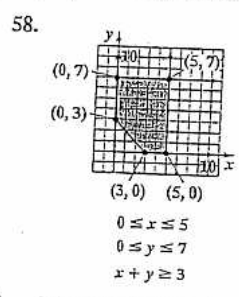
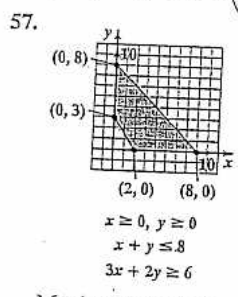
28. $\{(3, \sqrt{6}), (3, -\sqrt{6}), (-3, \sqrt{6}), (-3, -\sqrt{6})\}$ 29. $\{(2, 2), (-2, -2)\}$ 30. $\{(9, 6), (1, 2)\}$ 31. $\{(-3, -1), (1, 3)\}$ 32. $\left\{\left(\frac{1}{2}, 2\right), (-1, -1)\right\}$
 33. $\left\{\left(\frac{5}{2}, -\frac{7}{2}\right), (0, -1)\right\}$ 34. $\{(2, -3), (-2, -3), (3, 2), (-3, 2)\}$ 35. $\{(3, 1), (3, -1), (-3, 1), (-3, -1)\}$ 36. 8 m and 5 m 37. $(1, 6), (3, 2)$
 38. $x = 46$ and $y = 28$ or $x = 50$ and $y = 20$



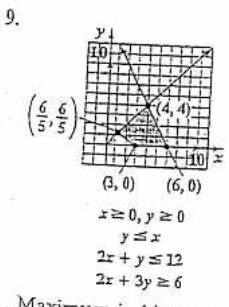
51. no solution



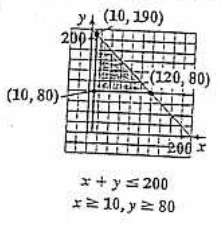
56. $(2, 2); 10; (4, 0); 8; \left(\frac{1}{2}, \frac{1}{2}\right); \frac{5}{2}; (1, 0); 2$; maximum value: 10; minimum value: 2



Maximum is 24 at $x = 0, y = 8$. Maximum is 33 at $x = 5, y = 7$.



60. a. $z = 500x + 350y$
 b. $x + y \leq 200; x \geq 10; y \geq 80$



d. $(10, 80): 33,000; (10, 190): 71,500; (120, 80): 88,000$
 e. 120; 80; 88,000

Maximum is 44 at $x = y = 4$.
 480 of model A and 240 of model B

In Exercises 1–6, write the first four terms of each sequence whose general term is given.

1. $a_n = 7n - 4$

2. $a_n = (-1)^n \frac{n+2}{n+1}$

3. $a_n = \frac{1}{(n-1)!}$

4. $a_n = \frac{(-1)^{n+1}}{2^n}$

5. $a_1 = 9$ and $a_n = \frac{2}{3a_{n-1}}$ for $n \geq 2$

6. $a_1 = 4$ and $a_n = 2a_{n-1} + 3$ for $n \geq 2$

7. Evaluate: $\frac{40!}{4!38!}$

In Exercises 8–9, find each indicated sum.

8. $\sum_{i=1}^5 (2i^2 - 3)$

9. $\sum_{i=0}^4 (-1)^{i+1} i!$

In Exercises 10–11, express each sum using summation notation. Use i for the index of summation.

10. $\frac{1}{3} + \frac{2}{4} + \frac{3}{5} + \dots + \frac{15}{17}$

11. $4^3 + 5^3 + 6^3 + \dots + 13^3$

In Exercises 12–15, write the first six terms of each arithmetic sequence.

12. $a_1 = 7, d = 4$

13. $a_1 = -4, d = -5$

14. $a_1 = \frac{3}{2}, d = -\frac{1}{2}$

15. $a_{n+1} = a_n + 5, a_1 = -2$

In Exercises 16–18, find the indicated term of the arithmetic sequence with first term, a_1 , and common difference, d .

16. Find a_6 when $a_1 = 5, d = 3$.

17. Find a_{12} when $a_1 = -8, d = -2$.

18. Find a_{14} when $a_1 = 14, d = -4$.

In Exercises 19–21, write a formula for the general term (the n th term) of each arithmetic sequence. Do not use a recursion formula. Then use the formula for a_n to find a_{20} , the 20th term of the sequence.

19. $-7, -3, 1, 5, \dots$

20. $a_1 = 200, d = -20$

21. $a_n = a_{n-1} - 5, a_1 = 3$

22. Find the sum of the first 22 terms of the arithmetic sequence: $5, 12, 19, 26, \dots$

23. Find the sum of the first 15 terms of the arithmetic sequence: $-6, -3, 0, 3, \dots$

24. Find $3 + 6 + 9 + \dots + 300$, the sum of the first 100 positive multiples of 3.

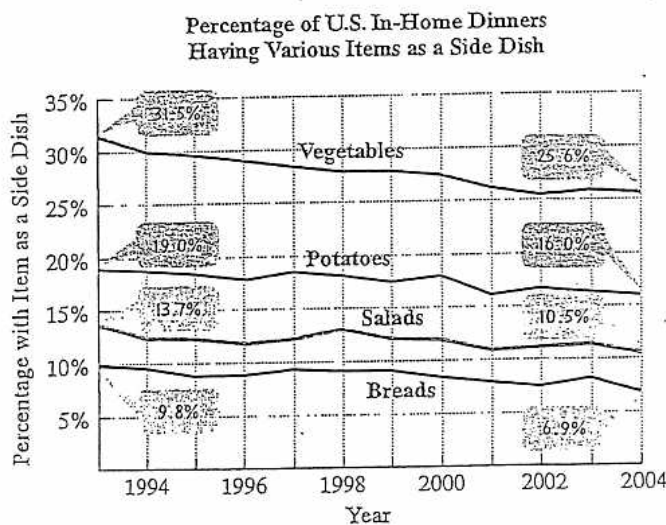
In Exercises 25–27, use the formula for the sum of the first n terms of an arithmetic sequence to find the indicated sum.

25. $\sum_{i=1}^{16} (3i + 2)$

26. $\sum_{i=1}^{25} (-2i + 6)$

27. $\sum_{i=1}^{30} (-5i)$

28. The graph shows the percentage of in-home dinners in the United States having various items as a side dish from 1993 through 2004.



Source: The NPD Group

In 1993, 31.5% of home dinners had vegetables as a side dish. On average, this decreased by approximately 0.54% per year since then.

- Write a formula for the n th term of the arithmetic sequence that describes the percentage of dinners that included vegetables n years after 1992.
 - Use the model to predict the percentage of dinners that will include vegetables by the year 2010.
 - Repeat parts (a) and (b) for the change in another one of the items from 1993 to 2004.
29. A company offers a starting salary of \$31,500 with raises of \$2300 per year. Find the total salary over a ten-year period.
30. A theater has 25 seats in the first row and 35 rows in all. Each successive row contains one additional seat. How many seats are in the theater?

In Exercises 31–34, write the first five terms of each geometric sequence.

31. $a_1 = 3, r = 2$

32. $a_1 = \frac{1}{2}, r = \frac{1}{2}$

33. $a_1 = 16, r = -\frac{1}{2}$

34. $a_n = -5a_{n-1}, a_1 = -1$

In Exercises 35–37, use the formula for the general term (the n th term) of a geometric sequence to find the indicated term of each sequence.

35. Find a_7 when $a_1 = 2, r = 3$.

36. Find a_6 when $a_1 = 16, r = \frac{1}{2}$.

37. Find a_5 when $a_1 = -3, r = 2$.

In Exercises 38–40, write a formula for the general term (the n th term) of each geometric sequence. Then use the formula for a_n to find a_8 , the eighth term of the sequence.

38. $1, 2, 4, 8, \dots$

39. $100, 10, 1, \frac{1}{10}, \dots$

40. $12, -4, \frac{4}{3}, -\frac{4}{9}, \dots$

41. Find the sum of the first 15 terms of the geometric sequence: $5, -15, 45, -135, \dots$

In Exercises 43–45, use the formula for the sum of the first n terms of a geometric sequence to find the indicated sum.

43. $\sum_{i=1}^6 5^i$

44. $\sum_{i=1}^7 3(-2)^i$

45. $\sum_{i=1}^5 2\left(\frac{1}{4}\right)^{i-1}$

In Exercises 46–49, find the sum of each infinite geometric series.

46. $9 + 3 + 1 + \frac{1}{3} + \dots$ 47. $2 - 1 + \frac{1}{2} - \frac{1}{4} + \dots$

48. $-6 + 4 - \frac{8}{3} + \frac{16}{9} - \dots$

49. $\sum_{i=1}^{\infty} 5(0.8)^i$

In Exercises 50–51, express each repeating decimal as a fraction in lowest terms.

50. $0.\overline{6}$ 51. $0.\overline{47}$

52. Projections for the U.S. population, ages 85 and older, are shown in the following table.

Year	2000	2010	2020	2030	2040	2050
Projected Population in millions	4.2	5.9	8.3	11.6	16.2	22.7

Actual 2000 population

Source: U.S. Census Bureau

- Show that the U.S. population, ages 85 and older, is projected to increase geometrically.
- Write the general term of the geometric sequence describing the U.S. population ages 85 and older, in millions, n decades after 2000.
- Use the model in part (b) to project the U.S. population, ages 85 and older, in 2080.

- A job pays \$32,000 for the first year with an annual increase of 6% per year beginning in the second year. What is the salary in the sixth year? What is the total salary paid over this six-year period? Round answers to the nearest dollar.
- You decide to deposit \$200 at the end of each month into an account paying 10% interest compounded monthly to save for your child's education. How much will you save over 18 years?
- A factory in an isolated town has an annual payroll of \$4 million. It is estimated that 70% of this money is spent within the town, that people in the town receiving this money will again spend 70% of what they receive in the town, and so on. What is the total of all this spending in the town each year?

In Exercises 61–62, evaluate the given binomial coefficient.

61. $\binom{11}{8}$

62. $\binom{90}{2}$

In Exercises 63–66, use the Binomial Theorem to expand each binomial and express the result in simplified form.

63. $(2x + 1)^3$

64. $(x^2 - 1)^4$

65. $(x + 2y)^5$

66. $(x - 2)^6$

In Exercises 67–68, write the first three terms in each binomial expansion, expressing the result in simplified form.

67. $(x^2 + 3)^8$

68. $(x - 3)^9$

In Exercises 69–70, find the term indicated in each expansion.

69. $(x + 2)^5$; fourth term 70. $(2x - 3)^6$; fifth term

In Exercises 71–74, evaluate each expression.

71. ${}_8P_3$

72. ${}_9P_5$

73. ${}_8C_3$

74. ${}_{13}C_{11}$

In Exercises 75–81, solve by the method of your choice.

- A popular brand of pen comes in red, green, blue, or black ink. The writing tip can be chosen from extra bold, bold, regular, fine, or micro. How many different choices of pens do you have with this brand?
- A stock can go up, go down, or stay unchanged. How many possibilities are there if you own five stocks?
- A club with 15 members is to choose four officers—president, vice-president, secretary, and treasurer. In how many ways can these offices be filled?
- How many different ways can a director select 4 actors from a group of 20 actors to attend a workshop on performing in rock musicals?
- From the 20 CDs that you've bought during the past year, you plan to take 3 with you on vacation. How many different sets of three CDs can you take?
- How many different ways can a director select from 20 male actors and cast the roles of Mark, Roger, Angel, and Collins in the musical *Rent*?
- In how many ways can five airplanes line up for departure on a runway?

The table shows differences in political ideology, by education, for a random sample of U.S. voters. (The ratios for each group's ideologies are sourced from voting patterns in the 2000 U.S. election. The frequencies shown are hypothetical.) Use the data to solve Exercises 82–87. Express probabilities as simplified fractions.

	Liberal	Moderate	Conservative
High School only	7	35	13
College	10	15	20

Find the probability that a randomly selected person from this group

- is liberal.
- is not conservative.
- is moderate or conservative.
- is conservative or attended college.

16. Among people with a conservative ideology, find the probability of selecting a person who attended high school only.
17. Among people who attended college, find the probability of selecting a person with a liberal ideology.

In Exercises 88–89, a die is rolled. Find the probability of

88. getting a number less than 5.
89. getting a number less than 3 or greater than 4.

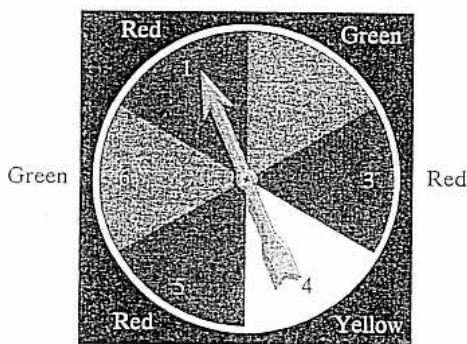
In Exercises 90–91, you are dealt one card from a 52-card deck.

Find the probability of

90. getting an ace or a king.
91. getting a queen or a red card.

In Exercises 92–94, it is equally probable that the pointer on the pinner shown will land on any one of the six regions, numbered through 6, and colored as shown. If the pointer lands on a borderline, spin again. Find the probability of

92. not stopping on yellow.
93. stopping on red or a number greater than 3.
94. stopping on green on the first spin and stopping on a number less than 4 on the second spin.



95. A lottery game is set up so that each player chooses five different numbers from 1 to 20. If the five numbers match the five numbers drawn in the lottery, the player wins (or shares) the top cash prize. What is the probability of winning the prize
 - a. with one lottery ticket?
 - b. with 100 different lottery tickets?
96. What is the probability of a family having five boys born in a row?
97. The probability of a flood in any given year in a region prone to floods is 0.2.
 - a. What is the probability of a flood two years in a row?
 - b. What is the probability of a flood for three consecutive years?
 - c. What is the probability of no flooding for four consecutive years?

Chapter 8 Review Exercises

1. $a_1 = 3; a_2 = 10; a_3 = 17; a_4 = 24$ 2. $a_1 = -\frac{3}{2}; a_2 = \frac{4}{3}; a_3 = -\frac{5}{4}; a_4 = \frac{6}{5}$ 3. $a_1 = 1; a_2 = 1; a_3 = \frac{1}{2}; a_4 = \frac{1}{6}$
4. $a_1 = \frac{1}{2}; a_2 = -\frac{1}{4}; a_3 = \frac{1}{8}; a_4 = -\frac{1}{16}$ 5. $a_1 = 9; a_2 = \frac{2}{27}; a_3 = 9; a_4 = \frac{2}{27}$ 6. $a_1 = 4; a_2 = 11; a_3 = 25; a_4 = 53$ 7. 65 8. 95
9. -20 10. $\sum_{i=1}^{15} \frac{i}{i+2}$ 11. $\sum_{i=2}^{15} i^3$ or $\sum_{i=1}^{10} (i+3)^3$ 12. 7, 11, 15, 19, 23, 27 13. -4, -9, -14, -19, -24, -29 14. $\frac{3}{2}, 1, \frac{1}{2}, 0, -\frac{1}{2}, -1$
15. -2, 3, 8, 13, 18, 23 16. $a_6 = 20$ 17. $a_{12} = -30$ 18. $a_{14} = -38$ 19. $a_n = 4n - 11; a_{20} = 69$ 20. $a_n = 220 - 20n; a_{20} = -180$
21. $a_n = 8 - 5n; a_{20} = -92$ 22. 1727 23. 225 24. 15, 150 25. 440 26. -500 27. -2325
28. a. $a_n = 32.04 - 0.54n$ b. 22.32% c. potatoes: $a_n = 19.27 - 0.27n; 14.41\%$; salads: $a_n = 13.99 - 0.29n; 8.77\%$; bread: $a_n = 10.06 - 0.26n; 5.38\%$
29. \$418,500 30. 1470 seats 31. 3, 6, 12, 24, 48 32. $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}$ 33. 16, -8, 4, -2, 1 34. -1, 5, -25, 125, -625
35. $a_7 = 1458$ 36. $a_6 = \frac{1}{2}$ 37. $a_5 = -48$ 38. $a_n = 2^{n-1}; a_8 = 128$ 39. $a_n = 100\left(\frac{1}{10}\right)^{n-1}; a_8 = \frac{1}{100,000}$
40. $a_n = 12\left(-\frac{1}{3}\right)^{n-1}; a_6 = -\frac{4}{729}$ 41. 17,936,135 42. $\frac{127}{8}$ 43. 19,530 44. -258 45. $\frac{341}{128}$ 46. $\frac{27}{2}$ 47. $\frac{4}{3}$ 48. $-\frac{18}{5}$
49. 20 50. $\frac{2}{3}$ 51. $\frac{47}{99}$ 52. a. $\frac{5.9}{4.2} \approx 1.4; \frac{8.3}{5.9} \approx 1.4; \frac{11.6}{8.3} \approx 1.4; \frac{16.2}{11.6} \approx 1.4; \frac{22.7}{16.2} \approx 1.4$ b. $a_n = 4.2(1.4)^n$ c. 62.0 million
53. \$42,823; \$223,210 54. \$120,113 55. \$9 $\frac{1}{3}$ million
56. $S_1: 5 = \frac{5(1)(1+1)}{2}; S_k: 5 + 10 + 15 + \dots + 5k = \frac{5k(k+1)}{2}; S_{k+1}: 5 + 10 + 15 + \dots + 5(k+1) = \frac{5(k+1)(k+2)}{2}; S_{k+1}$
can be obtained by adding $5(k+1)$ to both sides of S_k .
57. $S_1: 1 = \frac{4^1 - 1}{3}; S_k: 1 + 4 + 4^2 + \dots + 4^{k-1} = \frac{4^k - 1}{3}; S_{k+1}: 1 + 4 + 4^2 + \dots + 4^k = \frac{4^{k+1} - 1}{3}; S_{k+1}$ can be obtained by adding 4^k to both sides of S_k .
58. $S_1: 2 = 2(1)^2; S_k: 2 + 6 + 10 + \dots + (4k - 2) = 2k^2; S_{k+1}: 2 + 6 + 10 + \dots + (4k + 2) = 2k^2 + 4k + 2; S_{k+1}$ can be obtained by adding $4k + 2$ to both sides of S_k .
59. $S_1: 1 \cdot 3 = \frac{1(1+1)[2(1)+7]}{6}; S_k: 1 \cdot 3 + 2 \cdot 4 + 3 \cdot 5 + \dots + k(k+2) = \frac{k(k+1)(2k+7)}{6}; S_{k+1}: 1 \cdot 3 + 2 \cdot 4 + 3 \cdot 5 + \dots + (k+1)(k+3) = \frac{(k+1)(k+2)(2k+9)}{6}; S_{k+1}$ can be obtained by adding $(k+1)(k+3)$ to both sides of S_k .
60. $S_1: 2$ is a factor of 6; $S_k: 2$ is a factor of $k^2 + 5k; S_{k+1}: 2$ is a factor of $k^2 + 7k + 6; S_{k+1}$ can be obtained from S_k by rewriting $k^2 + 7k + 6$ as $(k^2 + 5k) + 2(k+3)$.
61. 165 62. 4005 63. $8x^3 + 12x^2 + 6x + 1$ 64. $x^8 - 4x^6 + 6x^4 - 4x^2 + 1$ 65. $x^5 + 10x^4y + 40x^3y^2 + 80x^2y^3 + 80xy^4 + 32y^5$
66. $x^6 - 12x^5 + 60x^4 - 160x^3 + 240x^2 - 192x + 64$ 67. $x^{16} + 24x^{14} + 252x^{12} + \dots$ 68. $x^9 - 27x^8 + 324x^7 - \dots$ 69. $80x^2$
70. $4860x^2$ 71. 336 72. 15,120 73. 56 74. 78 75. 20 choices 76. 243 possibilities 77. 32,760 ways 78. 4845 ways
79. 1140 sets 80. 116,280 ways 81. 120 ways 82. $\frac{17}{100}$ 83. $\frac{67}{100}$ 84. $\frac{83}{100}$ 85. $\frac{29}{50}$ 86. $\frac{13}{33}$ 87. $\frac{2}{9}$ 88. $\frac{2}{3}$ 89. $\frac{2}{3}$
90. $\frac{2}{13}$ 91. $\frac{7}{13}$ 92. $\frac{5}{6}$ 93. $\frac{5}{6}$ 94. $\frac{1}{6}$ 95. a. $\frac{1}{15,504}$ b. $\frac{25}{3876}$ 96. $\frac{1}{32}$ 97. a. 0.04 b. 0.008 c. 0.4096