2.2

Note Title 8/25/2006

Difference Quotient

The Expression

$$\frac{f(x+h)-f(x)}{h}$$

for $h \neq 0$ is called the difference quotient.

Ex Let $f(x) = x^2 - 3x + 2$

find $\frac{f(x+h)-f(x)}{h}$ for $h \neq 0$

Let's start by just finding the $f(x+h)$ part.

$$f(x+h) = (x+h)^2 - 3(x+h) + 2$$
$$= x^2 + 2hx + h^2 - 3x - 3h + 2$$

now let's combine this with the rest of the difference quotient.

$$\frac{f(x+h)-f(x)}{h} = \frac{x^2+2hx+h^2-3x-3h+2-(x^2-3x+2)}{h}$$

$$= \frac{2hx+h^2-3h}{h}$$

$$= \frac{h(2x+h-3)}{h}$$

$$= 2x+h-3$$

Piecewise Functions

(ex) Your cell phone company computes your bill according to the following formula.

$$B(m) = \begin{cases} 30 & 0 \le m < 100 \\ 30 + .25m & 100 \le m < 500 \\ 30 + .50m & m \ge 500 \end{cases}$$

where $B(m)$ is your bill and m is the minutes you used.

If you use 364 minutes how much will you owe?

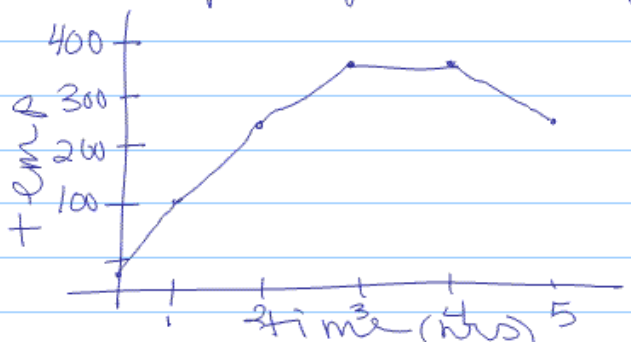
$$B(364) = 30 + .25(364)$$
$$= \boxed{\$121}$$

What if you used 800 min.

$$B(800) = 30 + .50(800)$$
$$= \$430$$

Increasing and Decreasing Functions

(ex) You're cooking! You decide to model the temperature of your food. You get this graph



this graph (and thus your food's temp.) is increasing between 0 and 3 hrs
Remains constant between 3 and 4 hrs and is decreasing between 4 and 5h

In math we say, the function is
increasing on the interval (0,3)
constant on the interval (3,4)
decreasing on the interval (4,5)

ex

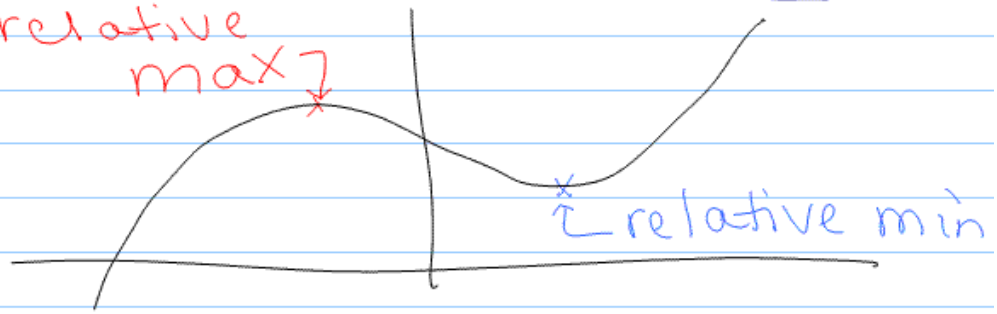
increasing $(-\infty, 2)$

decreasing $(2, \infty)$

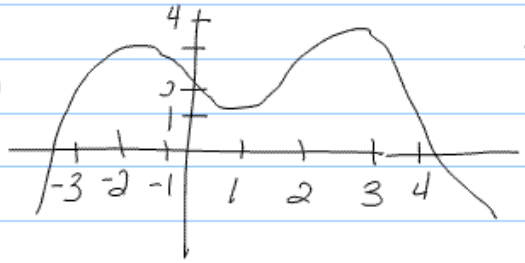
note: always go left to right!
always use x-coordinates!

Relative Maxima & Minima

ex



ex



f(x) has a relative min at $x=1$
the relative min is 1 (since $f(1)=1$)

f(x) has relative maximums at $x=-2$ and at $x=3$
the relative maximums are 3 and 4.

Even/Odd functions & Symmetry

A function f is <u>even</u> if
$f(-x) = f(x)$ for all x

A function f is <u>odd</u> if
$f(-x) = -f(x)$

note: Some functions are neither!

ex $f(x) = x^2$ is even because
$f(-x) = (-x)^2$
$= (-1\,x)^2 = (-1)^2(x^2) = x^2 = f(x)$
So $f(-x) = f(x)$

ex $f(x) = x^3$ is odd because
$f(-x) = (-x)^3 = (-1x)^3 = (-1)^3(x^3) = -x^3 = -f(x)$
So $f(-x) = -f(x)$

ex $f(x) = x^2 + 3x$ is neither

since

$f(-x) = (-x)^2 + 3(-x)$

$= x^2 - 3x$

but $f(x) = x^2 + 3x$ and

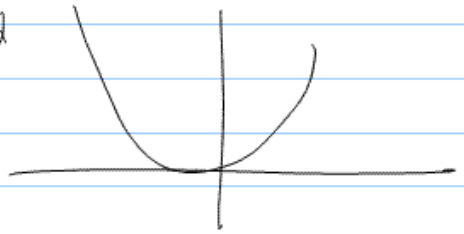
$-f(x) = -x^2 - 3x$

Try one: $f(x) = x^2 + 4$

ans. even.

Even functions are symmetric about the y-axis

ex $f(x) = x^2$



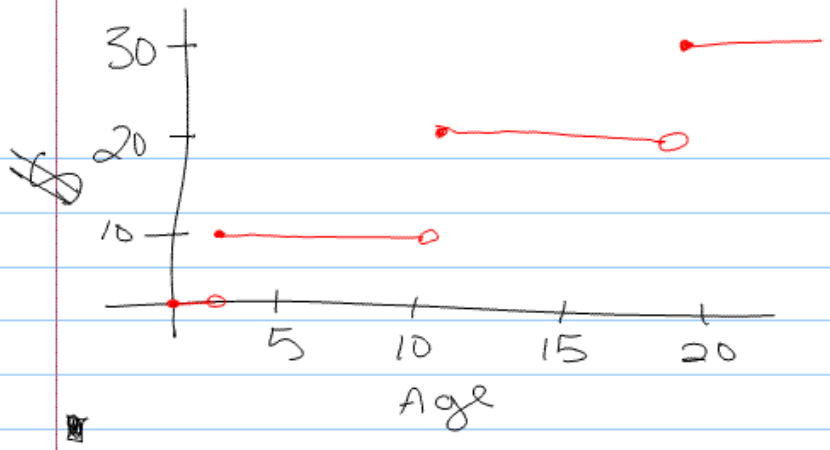
Odd functions are Symmetric about the origin.

ex $f(x) = x^3$

Step Functions

ex The price to get into a water park is as follows

Age	Price
$age < 2$	free
$2 \leq age < 10$	\$10
$10 \leq age < 18$	\$20
$age \geq 18$	\$30



Greatest Integer Function (floor func.)
denoted int(x) or $[\![x]\!]$

int(x) = the greatest integer that is less than or equal to x.
(idea: "round down")

ex) int(4.7) = 4 int(3) = 3