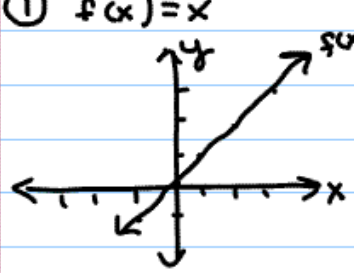
Section 2.5 Transformation of Functions

Note Title 1/9/2006

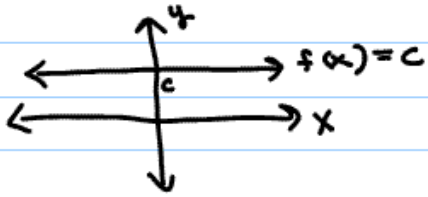
You need to know certain common functions so well you could draw them in your sleep!

① $f(x) = x$



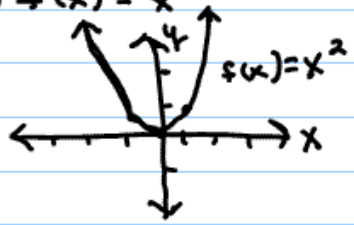
identity function

② $f(x) = c$, c is a constant



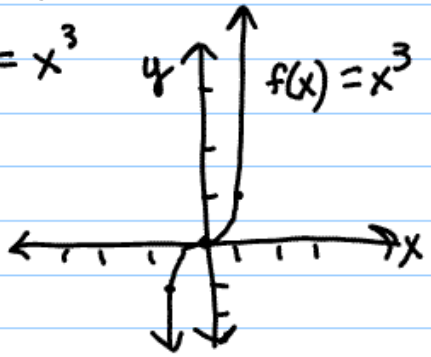
constant function

③ $f(x) = x^2$



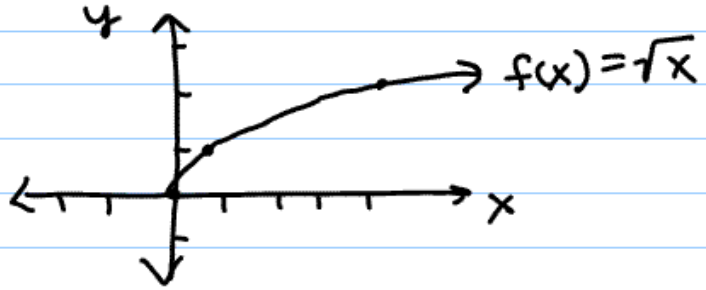
quadratic function

④ $f(x) = x^3$



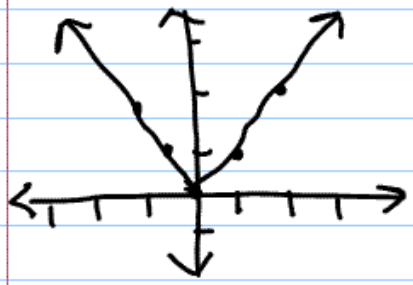
cubic function

⑤ $f(x) = \sqrt{x}$



square root function

⑥ $f(x) = |x|$



absolute value function

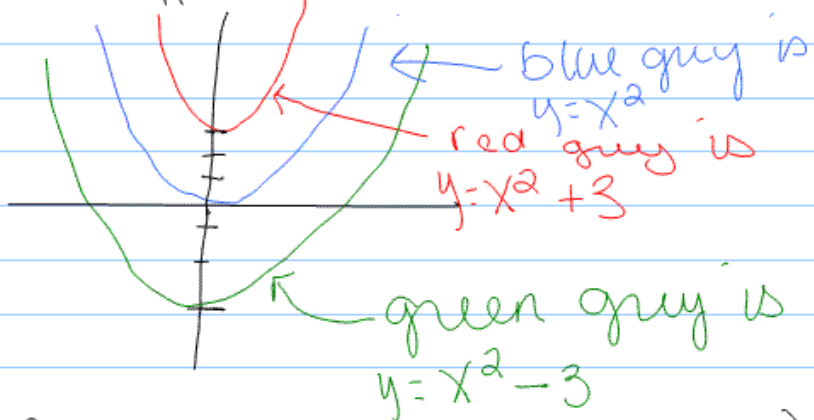
Now many more functions have graphs that are transformations of the above graphs, through shifting (up, down, left, or right), reflecting (about x or y-axis), and stretching or shrinking or combinations of transformations.

Vertical Shift

$y = f(x) + c$ shift up c units

$y = f(x) - c$ shift down c units

(ex)

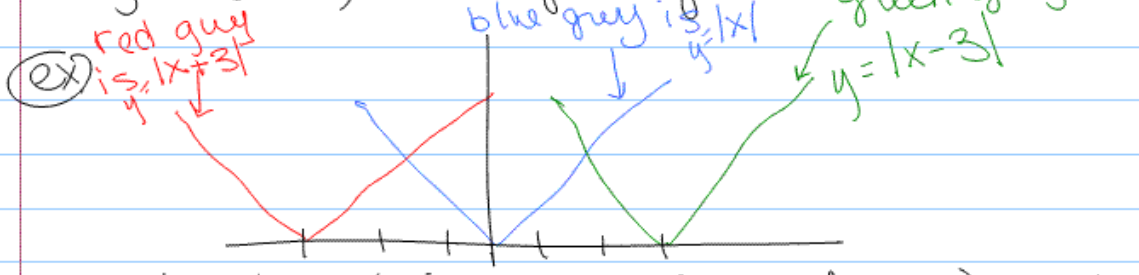


Horizontal Shifts (Careful of signs)

$y = f(x+c)$ shift left c

$y = f(x-c)$ shift right c

(ex)

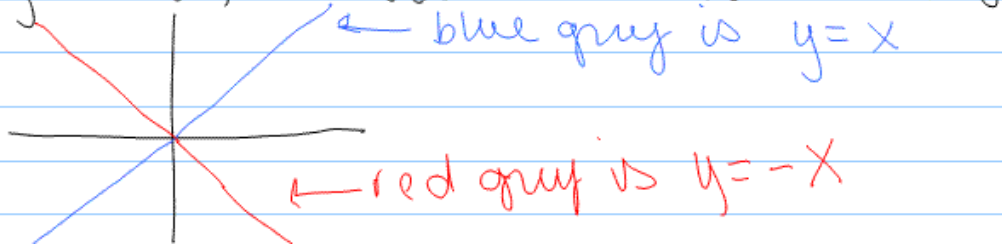


note: $|x+3|$ is in the form $f(x+c)$ but $|x|+3$ is in the form $f(x)+c$

Reflections

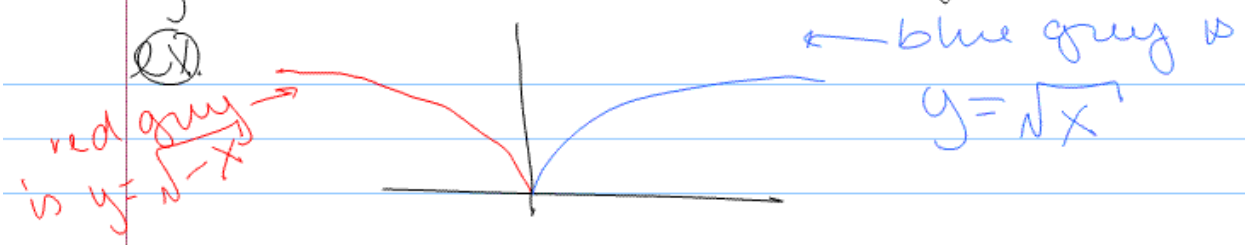
$y = -f(x)$ reflects about x-axis

(ex)



$y = f(-x)$ reflects about y-axis

ex)

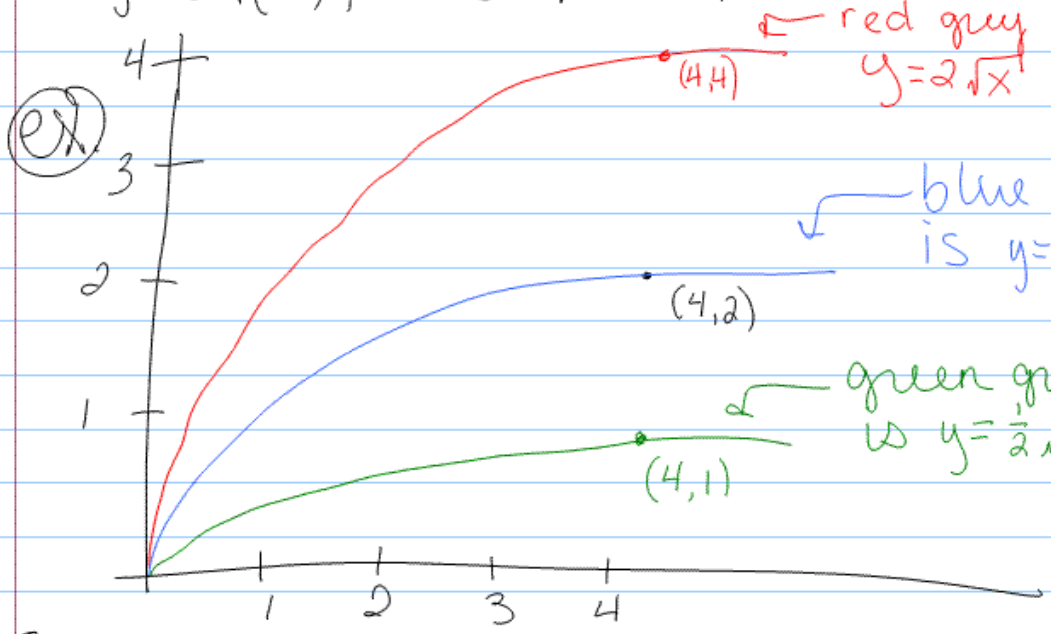


Vertical Stretching/Shrinking

$y = c \cdot f(x)$, $c > 1$ vertical stretch

$y = c \cdot f(x)$, $0 < c < 1$ vertical shrink

ex)



Don't worry about horizontal stretching/shrinking.

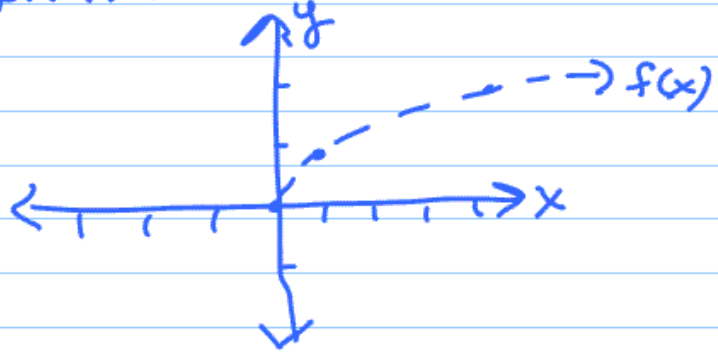
Important: combinations of functions must be performed in the following **order**:

1. Horizontal shifting
2. Vertical stretching or shrinking
3. Reflecting
4. Vertical shifting

Be sure you know how to use the table, **Summary of Transformations**, on page 250 (this may be something you would put in your note sheet for an exam).

Example: Graph $g(x) = 3\sqrt{x+1} - 2$

1. Start with the function you know:
   $f(x) = \sqrt{x}$ (square root guy)
   Graph it:

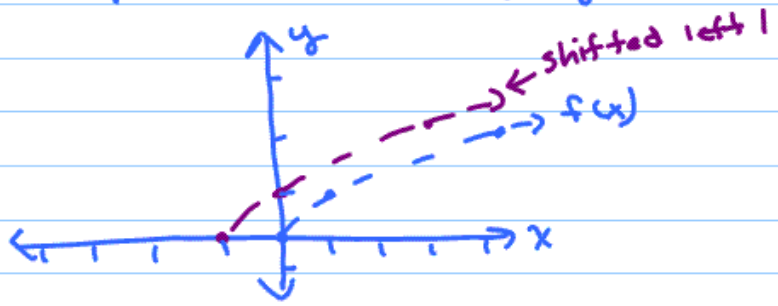


Now do transformations one at a time in correct order (see above)

2. $g(x) = 3\sqrt{x+1} - 2$

↑ horizontal shift left by 1

So take graph of $f(x)$ and shift all the points over (left) by 1:

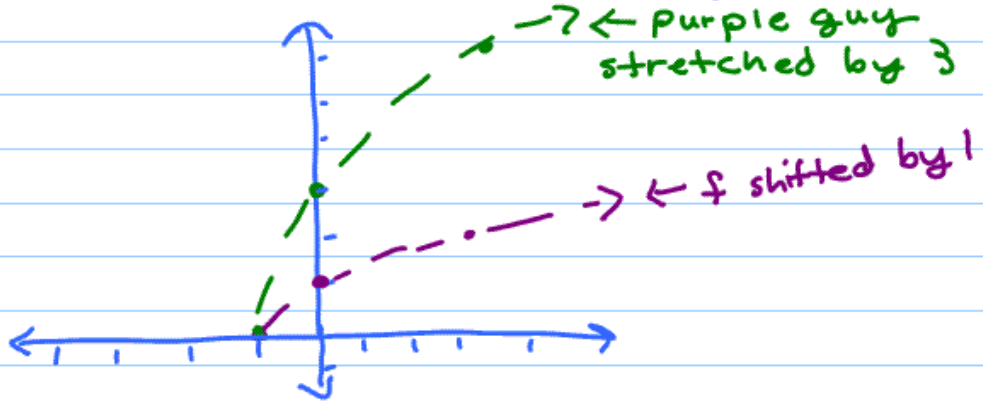


3. Now take new graph (purple guy) and do next transformation

$g(x) = 3\sqrt{x+1} - 2$

↑ vertical stretch by 3

So take graph from step 2 and multiply all the y-coordinates by 3:



4. The next transformation we would do is reflecting, but there isn't any reflecting in

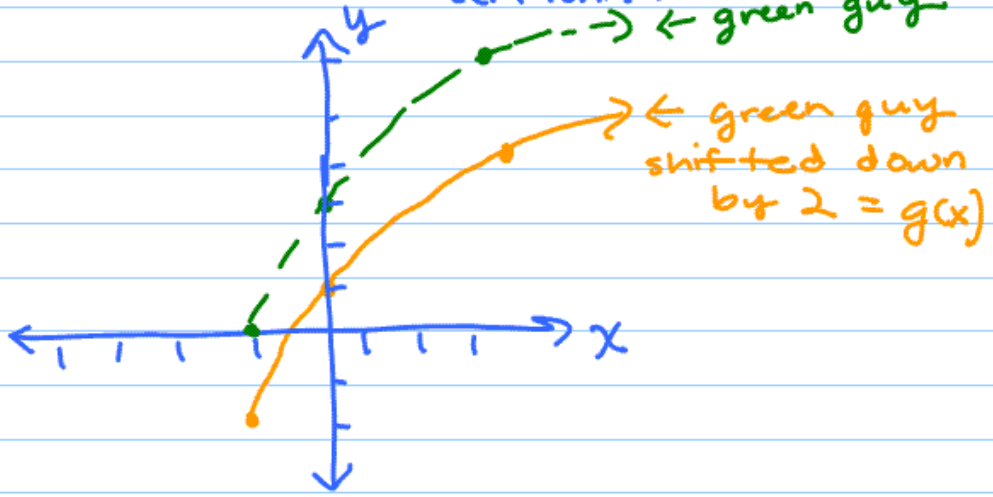
$$g(x) = 3\sqrt{x+1} - 2$$

so we don't have to do it

5. Now take graph from step 3 (green guy) and do final transformation

$$g(x) = 3\sqrt{x+1} - 2$$

vert. shift down 2



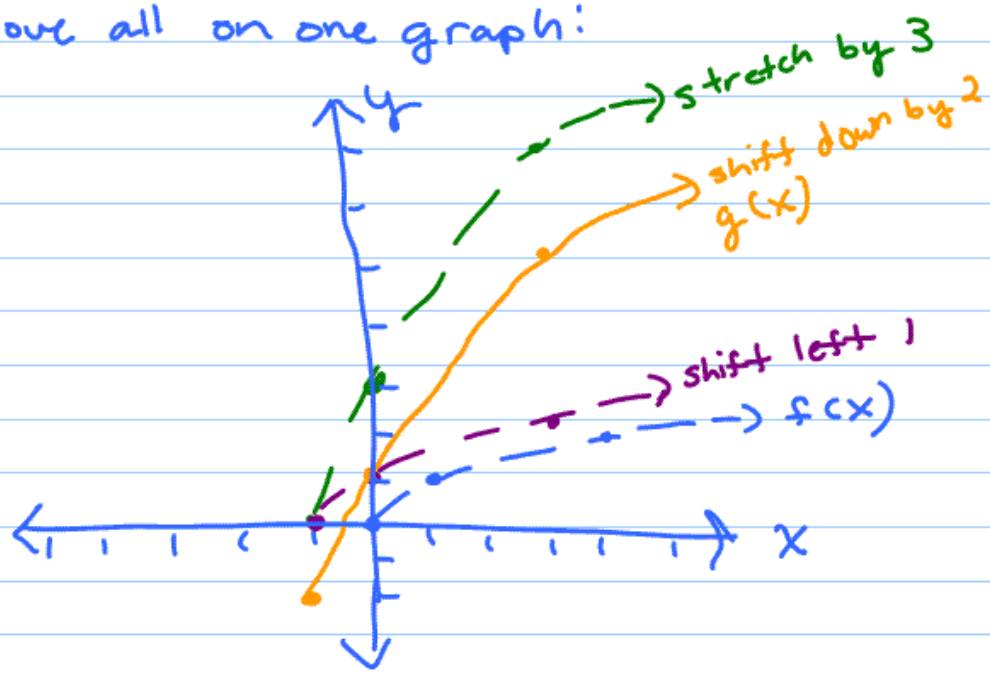
Now we are done! The orange guy above is the graph of

$$g(x) = 3\sqrt{x+1} - 2$$

Note: this example looks really long and painful but it's really not so bad.

You could do all these transformations on one graph and the more you do these, the quicker and easier they get:

Above all on one graph:



Try doing these graphs on your calculator!