Section 2.6 Combinations of Functions

Note Title 1/11/2006

Domain of a function

Remember: The domain of a function is a set of all possible input values ⇒ all values of x for which the function is defined.

note: exclude from the domain any # that
- divides by zero
- takes even root of a negative #.

Examples: Find the domain of the functions:

ⓐ $f(x) = \frac{1}{x+2}$

this function is undefined where the denominator is 0 ⇒ undefined for $x+2=0$

$x=-2$ → exclude from domain

Domain: $\{x \mid x \neq -2\}$

ⓑ $g(x) = \sqrt{3x-2}$

must have $3x-2 \geq 0$

$3x \geq 2$

$x \geq \frac{2}{3}$

domain: $\{x \mid x \geq \frac{2}{3}\}$

ⓒ $k(x) = x^2 + 3x - 8$

Any number we put in for x would be ok!

so, domain: all real numbers

Combining Functions

Since functions are usually like equations involving algebraic expressions, we can add, subtract, multiply, and divide functions the same way we do these operations on algebraic expressions.

Example: Let $f(x) = -2x^2 + 4$ and $g(x) = x - 1$

Then

(a) $(f+g)(x) = f(x) + g(x) = (-2x^2+4) + (x-1)$
$= -2x^2 + x + 3$

(b) $(f-g)(x) = f(x) - g(x) = (-2x^2+4) - (x-1)$
$= -2x^2 + 4 - x + 1$
$= -2x^2 - x + 5$

(c) $(f \cdot g)(x) = f(x) \cdot g(x) = (-2x^2+4)(x-1)$
$= -2x^3 + 4x + 2x^2 - 4$

(d) $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{-2x^2+4}{x-1}$ (can't simplify)

Note: The **domain** of a combination of functions f and g is all real numbers **common** to both domains (intersection of both domains).

Remember: $\frac{f(x)}{g(x)}$ must also have $g(x) \neq 0$.

Example: Given $f(x) = \sqrt{x}$ and $g(x) = \frac{1}{x-2}$

Find (a) $\left(\frac{f}{g}\right)x$ and (b) domain of $\frac{f}{g}$

(a) $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{\sqrt{x}}{x-2}$

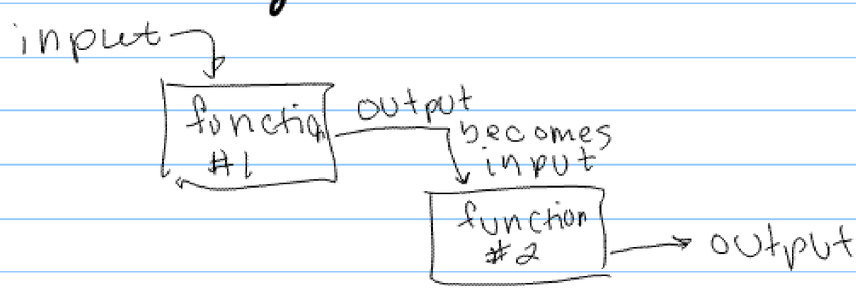
(b) domain of $f(x) = \sqrt{x}$ is all real numbers ≥ 0.

domain of $g(x) = \frac{1}{x-2}$ is all real numbers x, $x \neq 2$

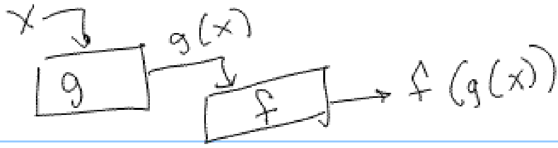
So domain of $\frac{f}{g}$ is all real numbers x, $x \geq 0$ and $x \neq 2$ OR domain: $\{x \mid x \geq 0, x \neq 2\}$

Composite Functions

Another way to combine functions:



In math notation



take output of g $\Rightarrow$ use as input of f

we write this as $f(g(x))$ or as $(f \circ g)(x)$

<u>Note</u>: The <u>composition</u> of f and g is denoted $f \circ g$ and

$$(f \circ g)(x) = f(g(x))$$

$\Rightarrow$ $(f \circ g)(x)$ means first do g, then f

The <u>domain</u> of $f \circ g$ is the set of input values x such that

1. x is in the domain of g and
2. g(x) is the domain of f

<u>Example</u>: $f(x) = \sqrt{x}$ and $g(x) = 2x+1$

(a) Find $(f \circ g)(x)$:

$$(f \circ g)(x) = f(g(x)) = f(2x+1) = \sqrt{2x+1}$$

(b) Find domain of $f \circ g$:

- Need x in domain of g (all real numbers)
- Need $g(x) = 2x+1$ in domain of f

So need $2x+1 \geq 0$

$$x \geq -\frac{1}{2}$$

so domain of $f \circ g = \{x \mid x \geq -\frac{1}{2}\}$

(c) Find $(f \circ g)(2) = \sqrt{2 \cdot 2+1} = \sqrt{5}$

(d) Find $(g \circ f)(x) = g(f(x))$

$= g(\sqrt{x})$

$= 2\sqrt{x} + 1$

<u>Note</u>: $(f \circ g)(x) \neq (g \circ f)(x)$ in this example

So you need to be careful of the order!