Section 2.7 Inverse Functions

Note Title 8/28/2006

Look at the function

$$f(x) = 2x + 3$$

notice what this function does.
it takes an x-value and

multiplies it by 2 then adds 3

Now look at the function

$$g(x) = \frac{x-3}{2}$$

notice what this function does.
it takes an x-value and

subtracts 3 then divides by 2

See how g "undoes" what f did (and vice-versa)

Note Title 1/21/2006

this is the idea of <u>inverse functions</u>

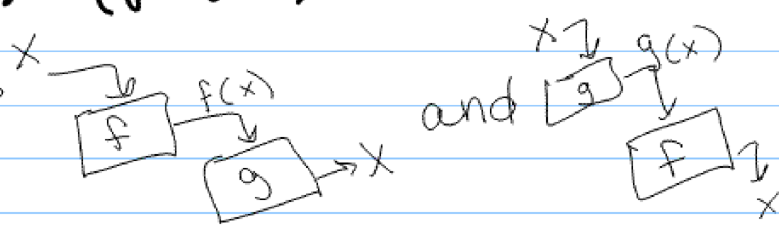
ex) Let's find the composite functions $f \circ g$ and $g \circ f$: (f & g from above)

$$(f \circ g)(x) = f(g(x)) = f\left(\frac{x-3}{2}\right)$$
$$= 2\left(\frac{x-3}{2}\right) + 3$$
$$= (x-3) + 3$$
$$= x$$

$$(g \circ f)(x) = g(f(x)) = g(2x+3)$$
$$= \frac{(2x+3)-3}{2}$$
$$= \frac{2x}{2}$$
$$= x$$

We have shown that
$$(f \circ g)(x) = (g \circ f)(x) = x$$

In pictures



Definition: Two functions f and g are inverses of each other if $(f \circ g)(x) = x$ for every x in the domain of g

and

$(g \circ f)(x) = x$ for every x in the domain of f.

Notation: we write f^{-1} (f inverse).

So we have $(f \circ f^{-1})(x) = (f^{-1} \circ f)(x) = x$

Note: For inverse functions f and g, the range of g must equal the domain of f and the range of f must equal the domain of g.

To show that functions are inverses, you must show that

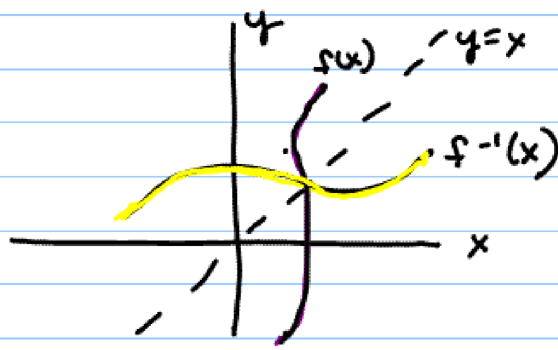
$(f \circ g)(x) = x$ and $(g \circ f)(x) = x$

Graphs of inverse functions

The graphs of f and its inverse, f^{-1}, are related.

If (x,y) is on the graph of f then (y,x) is on the graph of f^{-1}.

In other words, f^{-1} is a reflection of f over the line $y=x$.



Note: Not every function has an inverse.

A graph doesn't have an inverse if it doesn't pass the horizontal line test. because its reflection over the line $y=x$ won't pass the vertical line test so won't be a function.

See page 275 in your book

Finding an inverse of a function

Given a function $f(x)$, to find its inverse (if it exists):

1. Change the f(x) to a y.

2. Interchange x and y
3. Solve for y. Check that y is a function of x.
4. Change the y to $f^{-1}(x)$

Example: Find inverse of $f(x) = \frac{3x+4}{5}$

1. write $y = \frac{3x+4}{5}$

2. Interchange x and y:

$$x = \frac{3y+4}{5}$$

3. Solve for y:

$$x = \frac{3y+4}{5}$$

$$5x = 3y+4$$

$$5x-4 = 3y$$

$$\frac{5x-4}{3} = y$$

Is y a function of x? Yes

4. So, $f^{-1}(x) = \frac{5x-4}{3}$

notice: we can check by finding $f \circ f^{-1}$ and $f^{-1} \circ f$.

$$f \circ f^{-1} = f\left(\frac{5x-4}{3}\right)$$

$$= \frac{3\left(\frac{5x-4}{3}\right)+4}{5}$$

$$= \frac{(5x-4)+4}{5}$$

$$= \frac{5x}{5} = x \quad \checkmark$$

$$f^{-1}(f) = f^{-1}\left(\frac{3x+4}{5}\right)$$

$$= \frac{5\left(\frac{3x+4}{5}\right)-4}{3}$$

$$= \frac{(3x+4)-4}{3}$$

$$= \frac{3x}{3} = x \quad \checkmark$$