Section 2.8 Distance and Midpoint Formulas; Circles

Note Title 11/6/2005

You should know the
distance formula:

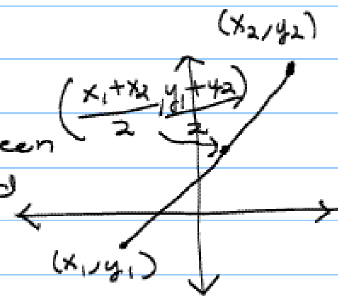
$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Note: it doesn't matter which point you call (x_1, y_1) and which you call (x_2, y_2)

You should also know the
midpoint formula:

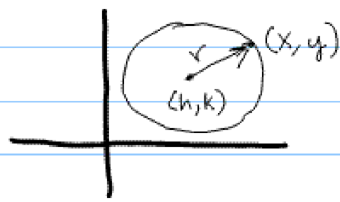
$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

midpoint between points (x_1, y_1) and (x_2, y_2)



Circles

Suppose you have a circle with center (h, k) and radius r



Pick any point (x, y) on the circle, then from the distance formula, the distance from (x, y) to (h, k) is r:

$$\sqrt{(x-h)^2 + (y-k)^2} = r$$

Now, squaring both sides gives:

$$\boxed{(x-h)^2 + (y-k)^2 = r^2}$$

This is the standard form of the equation of a circle with radius r and center (h, k).

Note: If the center of the circle is the origin $(0,0)$, the the above equation has $(h, k) = (0,0)$

so, we have $(x-0)^2 + (y-0)^2 = r^2$

giving $x^2 + y^2 = r^2$

Example: write the standard form of the equation of the circle with center $(2, -1)$ and radius $= 4$ and sketch a graph

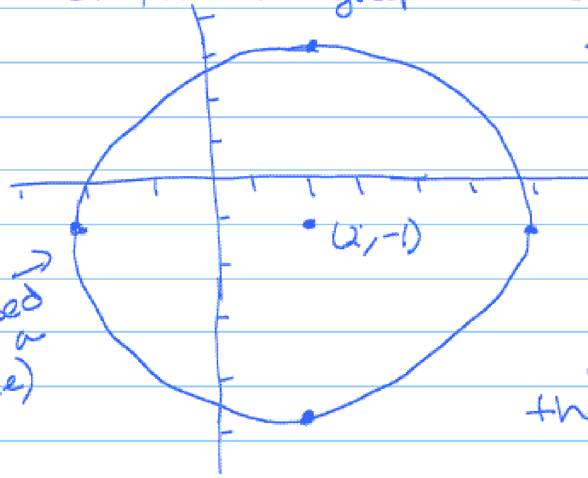
Solution: Given $(h,k) = (2,-1)$
$r = 4$

put in formula $(x-h)^2 + (y-k)^2 = r^2$

$(x-2)^2 + (y-(-1))^2 = 4^2$

standard form $\Rightarrow$ $(x-2)^2 + (y+1)^2 = 16$

Graph: 1st graph the center



(that's supposed to be a circle)

Since $r=4$, you can get points on the circle by going 4 units in any direction from the center

What if we expand the equation in above example: $(x-2)^2 + (y+1)^2 = 16$

$x^2 - 4x + 4 + y^2 + 2y + 1 = 16$ simplify

$x^2 + y^2 - 4x + 2y + 5 = 16$

$x^2 + y^2 - 4x + 2y - 11 = 0 \leftarrow$ general form

The general form of the equation of a circle is:

$$x^2 + y^2 + Dx + Ey + F = 0$$

Ex.: Suppose we have an equation in general form:

$$x^2+y^2-4x-12y-9=0$$

we need to write this equation in standard form in order to know the center and radius

How do we do this?

Answer: complete the square on x and y

Note: to review completing the square, see Section 1.5, pg. 134

or for a quick review go to the last page of notes!

Back to example: We have

$$x^2+y^2-4x-12y-9=0$$

$$(x^2-4x)+(y^2-12y\ )=9 \leftarrow$$ group x terms, y terms, put constant on right

$$(x^2-4x+4)+(y^2-12y+36)=9+4+36$$

complete the square on x

complete the square on y

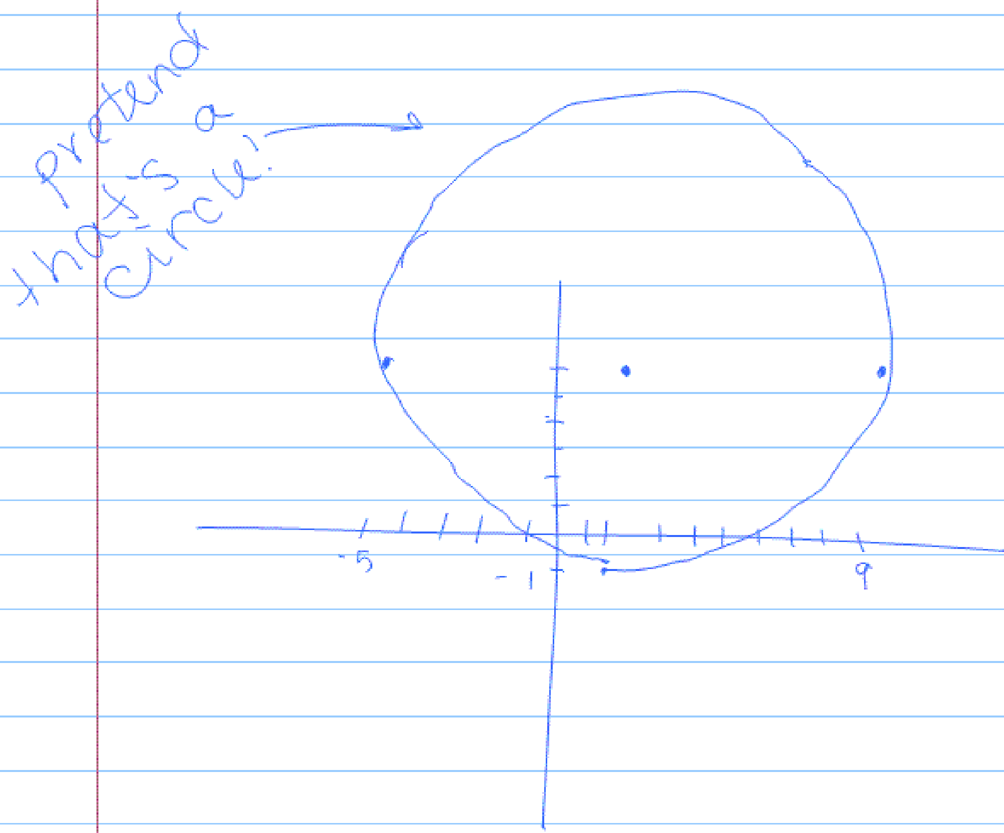
add 4 and 36 to both sides

$$\underbrace{(x-2)^2+(y-6)^2=49}$$ factor on left and add on right

Hey! This is in standard form!

circle w/ center (2,6) and radius = 7.

Now you could graph it



Quick Review of Completing the Square

ex) Use completing the square to solve $x^2+6x-2=0$

$x^2+6x=2$ move "c" over

$x^2+6x+(\frac{6}{2})^2=2+(\frac{6}{2})^2$ add $(\frac{b}{2})^2$ to both sides

$x^2+6x+9=11$

$(x+3)^2=11$ factor left side

$x+3=\pm\sqrt{11}$ use Square Root Property

$x+3=\sqrt{11}$ or $x+3=-\sqrt{11}$

$x=\sqrt{11}-3$ or $x=-\sqrt{11}-3$ solve for x

General Steps

1. Rewrite equation as $x^2+bx+c=0$ (may need to divide by "a" and/or rearrange terms)
2. move "c" to right side
3. add $(\frac{b}{2})^2$ to both sides
4. factor left side (always factors as $(x+\frac{b}{2})^2$)
5. take $\sqrt{}$ of left and $\pm\sqrt{}$ of right
6. Solve for x.