Section 3.1 Quadratic Functions

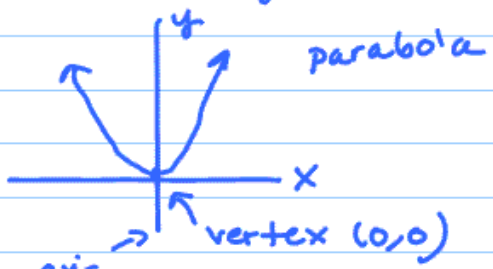
Note Title 1/21/2006

A function of the form $f(x) = ax^2 + bx + c$, $a \neq 0$, is called a **quadratic function**.

The graph of a quadratic function is called a **parabola**.

Example: ⓐ $f(x) = x^2$

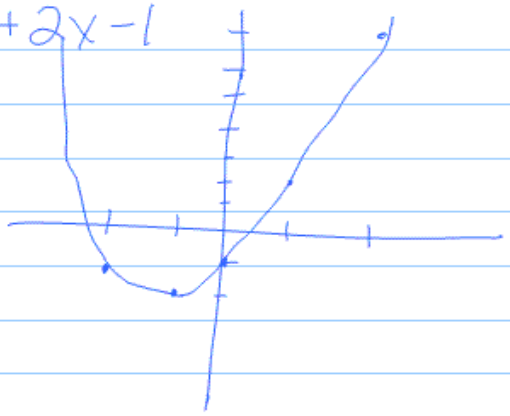
we've already seen its graph



axis of symmetry is the line $x = 0$

ⓑ $f(x) = x^2 + 2x - 1$

x	y
-2	-1
-1	-2
0	-1
1	2
2	7



vertex $(-1, -2)$

axis of symmetry $x = -1$

Another form of a quadratic equation looks like:

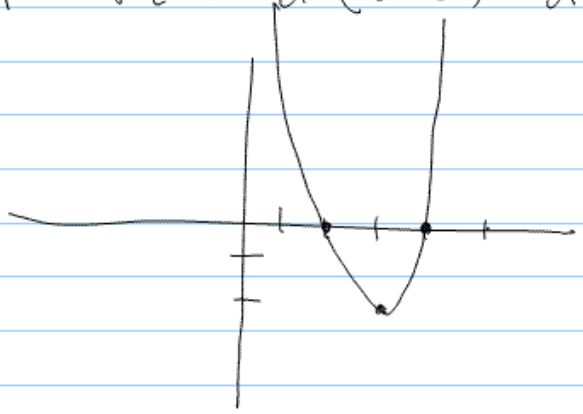
$f(x) = a(x-h)^2 + k$, $a \neq 0$

This is called the <u>standard form</u> of a quadratic equation.

<u>Note</u>: Given a quadratic equation in standard form, we can say:

1. Its vertex is at (h, k)
2. Its axis of symmetry is the line $x = h$
3. It opens up ↑ if $a > 0$ and down ↓ if $a < 0$
4. The x-intercepts (if any) occur where $f(x) = 0$ (where $y = 0$)
5. The y-intercept occurs at $f(0)$ (when $x = 0$)

(ex) Graph $f(x) = 2(x-3)^2 - 2$



1. vertex $(3, -2)$
2. axis of symm. is $x = 3$
3. $a = 2 > 0$ so opens up
4. x-int are $x = 2$ and 4
5. y-int: $y = 16$

Suppose an equation is not in standard form. How do you graph it?

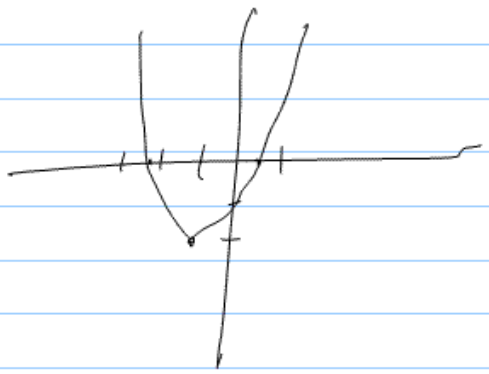
Given $f(x) = ax^2 + bx + c$

if you complete the square on f, you find that the vertex occurs when

$$x = -\frac{b}{2a}$$

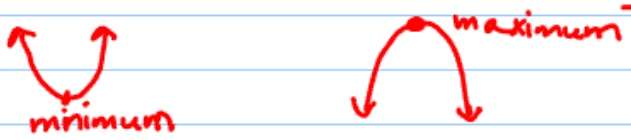
The other information you can find in the same way as in instructions 2 through 5 above.

Ex $f(x) = x^2 + 2x - 1$



1. vertex $(-1, -2)$
2. axis of symm. is $x = -1$
3. $a = 1 > 0$ so opens up
4. x-int are $x = \pm\sqrt{2} - 1$
5. y-int: $y = -1$

Remember that the maximum or minimum of a quadratic function occurs at its vertex.



You can use this information to solve certain applications.

(ex) Among all pairs of numbers whose difference is 12, find a pair whose product is as small as possible.

Let numbers be x and y.

$\Rightarrow y - x = 12$

$\Rightarrow y = 12 + x$

minimize $x \cdot y = x(12+x)$

$= 12x + x^2$

note: vertex is a minimum because the parabola opens up!

to minimize $x^2 + 12x$ you want to find the vertex

x-coordinate of vertex is $-b/2a = -6$

So $x = -6$

$\Rightarrow y = 12 + -6 = 6$

So -6 and 6 are the pair of numbers whose difference is 12 and product is smallest.

General Approach to Solving Maximum/Minimum Problems.

1. Read Carefully! Assign variables to unknowns and decide what is being maximized/minimized.

2. Write a function for quantity being max-/minimized

(a.k.a. thing to be maxed = _____
thing to be min-ed = _____)

Check that your parabola opens the right way:
up for min.
down for max.

3. find the vertex.

4. Answer the question asked.