Section 3.2 Polynomial Functions

Note Title 1/30/2006

<u>Recall</u>: The <u>degree of a polynomial</u> is the highest degree of any of its terms.

We already know how to graph the following polynomials:

function	degree	graph
$f(x)=c$	0	horiz. line
$f(x)=ax+b$	1	line
$f(x)=ax^2+bx+c$	2	parabola

What about <u>higher degree</u> polynomials?

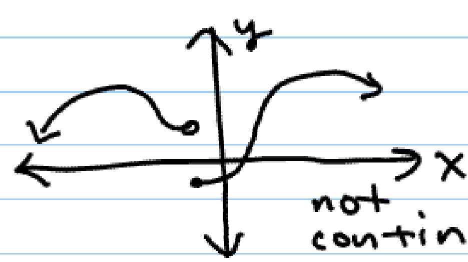
<u>Ex.</u>: $x^4-6x^3+9x^2$ (degree $=4$)

Such polynomials are difficult to graph just by plotting points.

We know some other <u>features</u> and look for other <u>clues</u> to help in graphing:

Polynomial functions are <u>smooth</u> and <u>continuous</u> $\Rightarrow$ no break or sharp corners

continuous



not continuous

Leading Coefficient Test

Given a polynomial of degree n:

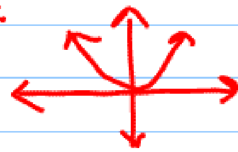
$$f(x) = a_n x^n + a_{n-1}x^{n-1} + \dots + a_1 x + a_0$$

Look at the leading coefficient, a_n and degree, n, to determine end behavior of graph

A.) If n is even → graph ends in same direction.

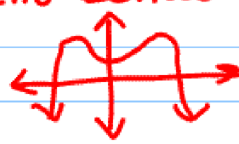
⇒ If $a_n > 0$ End behavior: ↑ ↑

Example:



⇒ If $a_n < 0$ End behavior: ↓ ↓

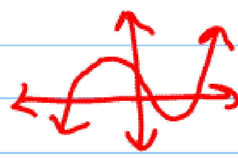
Example:



B.) If n is odd → graph ends in opposite directions

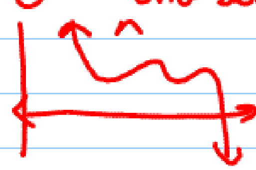
⇒ If $a_n > 0$ End behavior: ↓ ↑

Example:



⇒ If $a_n < 0$ End behavior: ↑ ↓

Example:



Example: $f(x) = x^3 - x$. Determine end behavior.

Leading coeff. $= 1 > 0$

Degree $= 3$ odd

End behavior: $\downarrow \quad \uparrow$

The graph looks like this:

Another helpful clue in graphing are the zeros!

f has at most n zeros (or roots)

Recall: A zero of a function is a solution to $f(x)=0$ or a point at which the graph crosses the x-axis (x-intercepts).

Finding the zeros helps in graphing

Example: Find all zeros of

$f(x) = x^3 + 2x^2 + x$

We know there are at most 3 zeros

Set $f(x)=0$ and solve for x:

$x^3 + 2x^2 + x = 0$

solve for x by factoring

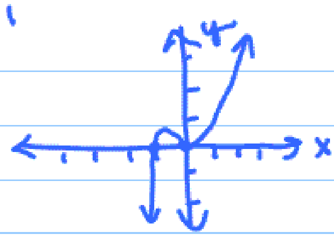
$x(x^2+2x+1) = 0$

$x(x+1)^2 = 0$

So, $x=0$ OR $x+1=0$
$x=-1$

Zeros: 0 and -1

Graph looks like:



Intermediate Value Theorem

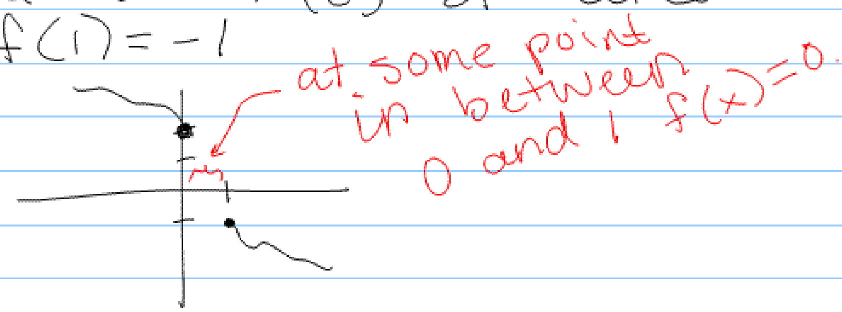
If $f(x)$ is a Polynomial and $f(a)$ & $f(b)$ have different signs then there exists a value c for $a<c<b$ such that $f(c)=0$

idea: if $f(a)$ is above the x-axis and $f(b)$ is below the x-axis at some point the graph hit the x-axis.

ex. Let $f(x)=x^3-4x^2+2$

notice that $f(0)=2$ and $f(1)=-1$

So



General Strategy for graphing polynomial functions:

1. Use Leading Coeff. Test to look at end behavior
2. Find zeros (x-intercepts)
3. Find y-intercept by finding $f(0)$
4. Check symmetry

   $f(-x) = f(x)$ y-axis symmetry (even)

   $f(-x) = -f(x)$ origin symmetry (odd)
5. Graph a few extra points to check it.

Note: Don't worry too much about multiplicity and turning points in graphing these polynomials.