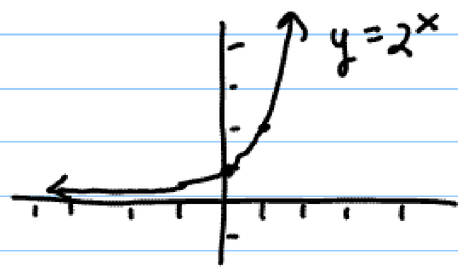
# Section 4.2 Logarithmic Functions

Note Title 2/26/2006

Consider again the graph of the exponential function: $f(x) = 2^x$ oR

$y = 2^x$




Does this function have an inverse?

Yes → it passes the horizontal line test

Let's try to find the inverse:

1. Interchange x and y:

$x = 2^y$

2. Solve for y:

$y = ???$

⇒ wait a second, how do we do that?

We introduce new notation so we can write this inverse function as $y =$ something

**Notation:** $y = \log_2 x$ means $2^y = x$

**Definition:** The function $y = \log_b x$ is the logarithmic function with base $b$, where $x > 0$, $b > 0$, and $b \neq 1$

$y = \log_b x$ is the same as $b^y = x$

ex) $y = \log_2 8 \Rightarrow 8 = 2^y$ so $y = 3$

$y = \log_4 16 \Rightarrow 4^y = 16$ so $y = 2$

$\log_3 \frac{1}{9} = ? \Rightarrow 3^? = \frac{1}{9}$, $3^{-2} = \frac{1}{9}$

so $\log_3 \frac{1}{9} = -2$

$\log_8 2 = ? \Rightarrow 8^? = 2$, $8^{1/3} = 2$

so $\log_8 2 = 1/3$

Basic Properties of Logarithms (come from props. of exponents):

If $b$ is a real number, $b > 0$ and $b \neq 1$, then

1. $\log_b b = 1$ $\quad (b^1 = b)$
2. $\log_b 1 = 0$ $\quad (b^0 = 1)$
3. $\log_b b^x = x$ $\quad (b^x = b^x)$
4. $b^{\log_b x} = x$

Note: Props. 3 and 4 come from the fact that the exponential function and logarithmic functions are inverses and so "undo" each other.

If $f(x) = b^x$, then $f^{-1}(x) = \log_b x$

So, $(f^{-1} \circ f)(x) = f^{-1}(f(x)) = f^{-1}(b^x)$

$= \log_b b^x = x$ (Prop. 3)

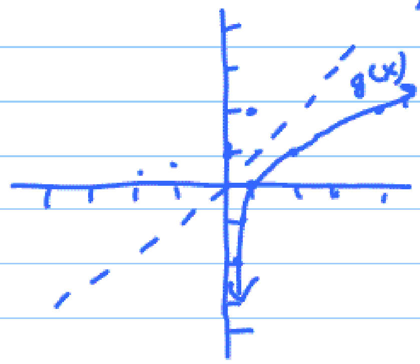
and $(f \circ f^{-1})(x) = f(f^{-1}(x))$

$= f(\log_b x)$

$= b^{\log_b x} = x$ (Prop. 4)

## Graphs of logarithmic functions

Example: Graph $g(x) = \log_2 x$
$x = 2^y$




| $x$ | $y = g(x)$ |
|---|---|
| 1 | 0 |
| 2 | 1 |
| 4 | 2 |
| $\frac{1}{2}$ | $-1$ |
| $\frac{1}{4}$ | $-2$ |

Note: $g(x)$ is a reflection of across the line $y = x$ (If $(x,y)$ is on $f$, then $(y,x)$ is on $g$).

General characteristics of the graph of $f(x) = \log_b x$:

1. $x$-intercept is 1
2. No $y$-intercept
3. Increases from left to right for $b > 1$
   Decreases from left to right for $0 < b < 1$
4. Domain: all real numbers s.t. $x > 0$
5. Range: all real numbers

Note: We can do transformations of logarithmic functions in the same way we did before.

Finding domain of log functions:

Remember: you can only find logarithms of positive values

Example: Find domain of
$f(x) = \log_5(x+6)$

Solution: need $x+6>0$
$x>-6$

domain: $\{x \mid x>-6\}$

Common Logarithms

common logarithm is log with base 10

Notation: we write $\boxed{f(x) = \log x}$

to mean
$f(x) = \log_{10} x$

(if no base is expressed, then it is log base 10 or common log)

Look for your $\boxed{\log x}$ key on your calculator → this is common log (base 10)

Natural Logarithms

We saw the natural exponential function $f(x) = e^x$

Its inverse is $f(x) = \log_e x$ ⇒ called the natural logarithmic function

Notation: we write $\boxed{f(x) = \ln x}$ to mean $f(x) = \log_e x$

Look for the $\boxed{\ln x}$ key on your calculator → this is natural log (base e)

Properties of natural log and common log are the same for logs in general:

| General | Common | Natural |
|---|---|---|
| 1. $\log_b b = 1$ | 1. $\log 10 = 1$ | 1. $\ln e = 1$ |
| 2. $\log_b 1 = 0$ | 2. $\log 1 = 0$ | 2. $\ln 1 = 0$ |
| 3. $\log_b b^x = x$ | 3. $\log 10^x = x$ | 3. $\ln e^x = x$ |
| 4. $b^{\log_b x} = x$ | 4. $10^{\log x} = x$ | 4. $e^{\ln x} = x$ |