Section 4.3 Properties of Logarithms

Note Title 2/27/2006

Important: Properties of logs come from the fact that logarithms are exponents and from the properties of exponents.

Recall certain properties of exponents:

1. $b^m \cdot b^n = b^{m+n}$
2. $\frac{b^m}{b^n} = b^{m-n}$

These props. of exponents lead to following:

Properties of Logarithms

1. $\log_b(M \cdot N) = \log_b M + \log_b N$
2. $\log_b\left(\frac{M}{N}\right) = \log_b M - \log_b N$
3. $\log_b M^p = p \log_b M$

Proof of prop. 1:

Let $x = \log_b M$ and $y = \log_b N$

then $b^x = M$ and $b^y = N$

so, $\log_b(MN) = \log_b(b^x \cdot b^y)$ (exponential notation)

$= \log_b(b^{x+y})$ (substitute) (prop. of exponents)

$= x + y$ (prop. of logs)

$= \log_b M + \log_b N$ (substitute)

QED

⇒ Proofs of Props. 2 and 3 are similiar

Important: $\log_b(M+N) \neq \log_b M + \log_b N$

ex) $\log_5(5x^2) = \log_5 5 + \log_5 x^2$

$= 1 + 2\log_5 x$

this is called expanding the log.

We can also use the above properties to go in the other direction $\Rightarrow$ write expanded expression as log of single quantity (condensing)

ex) $3\log x - \log 8 = \log \frac{x^3}{8}$

The change-of-Base Property

Calculators give us values of common log (log key) and natural log (ln key). But how do we find logs w/other bases?

We use:

change-of-Base Property:

$$\log_b M = \frac{\log_a M}{\log_a b}$$

, a, b, M are pos. real numbers and $b \neq 1$ and $a \neq 1$

base b (of $\log_b M$); base a (of $\log_a M$ and $\log_a b$)

We can use the change-of-base prop. to change a log with one base into a log with another base (we usually use base 10 or base e so we can use our calculator to simplify)

Example: Estimate $\log_4 30$ using common logs

Solution: $\log_4 30 = \frac{\log 30}{\log 4}$

↑ change to base 10 using change-of-base formula

$\approx \frac{1.4771}{.6021}$ use calculator

≈ 2.4534