Section 4.4 Exponential + Logarithmic Equations

Note Title 3/6/2006

Solving Exponential Equations

An exponential equation is an equation having a variable in an exponent

Examples:
(a) $6^x = 36$
(b) $2^x = 11$
(c) $2e^{2x} = 8$

Solve for the variable:

(a) $6^x = 36$

$6^x = 6^2$ ⇐ each side has same base

so, $x = 2$

In general, if $b^M = b^N$ the $M = N$ (same base)

what if both sides of an equation cannot be written with same base?

Ex: (b) $2^x = 11$

We want to "undo" the exponential to solve for x ⇒ since logs are inverses of exponentials ⇒ take log of both sides ⇒ use $\ln$ because can find values on calculator

solution: $2^x = 11$

$\ln 2^x = \ln 11$

$x \ln 2 = \ln 11$ (prop. of logs)

$x = \ln 11 / \ln 2 \approx 3.46$ (on calculator)

* **To solve exponential equations:**

1. Isolate the exponential expression
2. Take ln of both sides
3. Simplify using either $\ln b^x = x \ln b$ OR $\ln e^x = x$ ↑ since $\ln e = 1$
4. Solve for x.

Examples: Solve

(a) $e^x = 14$

$\ln e^x = \ln 14$

$x = \ln 14$ (can find on calculator)

(b) $2e^{2x} = 8$

$e^{2x} = 4$ (isolate exponential expression)

$\ln e^{2x} = \ln 4$

$2x = \ln 4$

$x = \frac{\ln 4}{2}$

(c) $3^{1-2x} - 1 = 1$

$3^{1-2x} = 2$ (isolate exponential expression)

$\ln 3^{1-2x} = \ln 2$

$(1-2x)\ln 3 = \ln 2$

$1 - 2x = \frac{\ln 2}{\ln 3}$

$-2x = \frac{\ln 2}{\ln 3} - 1$

$x = -\frac{1}{2}\left(\frac{\ln 2}{\ln 3} - 1\right)$ (find on calculator)

Solving Logarithmic Equations

Equation involving logs

Example: ⓐ Solve $\log_3(x+1)=5$

⇒ Rewrite in exponential notation

$$3^5 = x+1$$
$$243 = x+1$$
$$242 = x$$

ⓑ Solve $\log_2 x - \log_2 3 = 1$

First, write as single log using props. of logs

$$\log_2 x - \log_2 3 = 1$$
$$\log_2\left(\frac{x}{3}\right) = 1 \quad \text{quotient rule}$$
$$2^1 = \frac{x}{3} \quad \text{write in exponential notation}$$
$$2 = \frac{x}{3}$$
$$x = 6$$

* Solving Logarithmic Equations

1. use props. of logs to rewrite the equation as an equation involving a single logarithmic expression
2. Rewrite the equation in exponential form.
3. solve for x.

Example: solve $\log_4 x = 2 - \log_4(x+6)$

$\log_4 x + \log_4(x+6) = 2$

$\log_4[(x)(x+6)] = 2$ (Product rule)

$\log_4(x^2+6x) = 2$

$x^2+6x = 4^2$ (exponential form)

$x^2+6x = 16$

$x^2+6x-16=0$ (solve for x)

$(x+8)(x-2)=0$

~~$x=-8$~~ OR $x=2$

↑ can't take log of a negative number

See Examples 8-10, pg. 413, for some Applications using exponential and logarithmic expressions