Note Title 3/7/2006

Section 5.1 Systems of Linear Equations in Two Variables

Recall: An equation of the form $ax+by=c$ is a **linear equation**. Its graph is a straight line.

Suppose we have **two** linear equations:

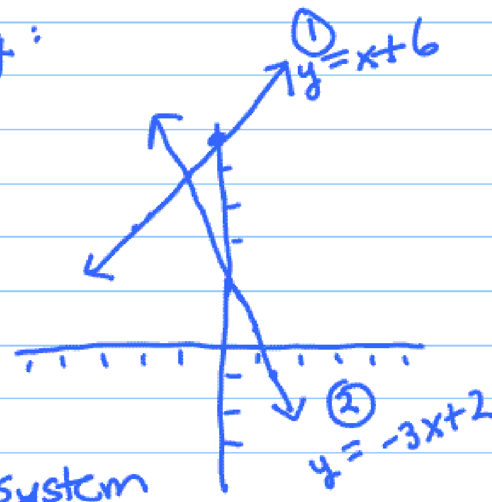
Example: $\begin{cases} -x+y=6 \\ 6x+2y=4 \end{cases}$ ⟵ System of linear equations

A **solution** to the system is an ordered pair (x,y) that is a solution to **both equations**.

Sometimes we can find the solution by **graphing** the lines:

Example: solve by graphing:

$$\begin{cases} -x+y=6 & (1) \\ 6x+2y=4 & (2) \end{cases}$$



Rewrite in slope-intercept form:
(1) $y=x+6$
(2) $2y=-6x+4$
$y=-3x+2$

The point $(-1,5)$ is on both graphs $\Rightarrow$ it is the only solution to the system

Check that $(-1,5)$ is a solution:

Note: You must check the solution in **both** original equations

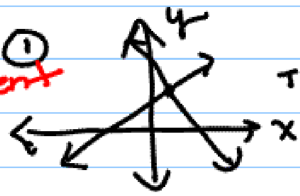
$-x+y=6$ ①	
$-(-1)+5$	
$1+5$	
6	6 ✓

$6x+2y=4$ ②	
$6(-1)+2(5)$	
$-6+10$	
4	4 ✓

There are 3 possibilities for the solutions to 2 linear equations:

different slopes

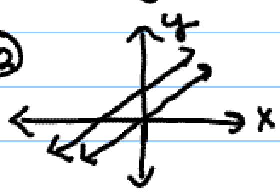
(1)



The lines intersect in one point → one solution

same slope

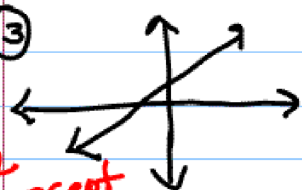
(2)



The lines are parallel → no solutions (inconsistent system)

same slope, same y-intercept

(3)



The lines are the same → every point on the line is a solution → infinite solutions (consistent system)

Sometimes it's difficult to solve by graphing → can't read the intersection point accurately

Another method:

<u>Substitution method</u>

Substitute for one variable in one of the equations

<u>Example:</u> (same as above)

Solve by substitution:

$$\begin{cases} -x+y=6 & (1) \\ 6x+2y=4 & (2) \end{cases}$$

Solve for y in (1): $y=x+6$

Substitute for y in (2):

$$6x+2(x+6)=4$$

Now solve for x:

$$6x+2x+12=4$$
$$8x+12=4$$
$$8x=-8$$
$$x=-1$$

Substitute $x=-1$ into either original equation to solve for y:

$$-x+y=6 \quad (1)$$
$$-(-1)+y=6$$
$$1+y=6$$
$$y=5$$

Solution: $(-1, 5)$ check it!

The substitution method may introduce icky fractions, so we have a third method:

Addition Method

Example: $\begin{cases} 2x-3y=0 & (1) \\ -4x+3y=-1 & (2) \end{cases}$

since the right side and left side of (1) are equal, we can add each side to (2):

add
$$\begin{array}{r} 2x-3y=0 \\ \underline{-4x+3y=-1} \\ -2x+0y=-1 \\ -2x=-1 \\ x=\frac{1}{2} \end{array}$$

Now substitute $x=\frac{1}{2}$ into either equation to find y:

$$\begin{array}{r} (1)\ 2x-3y=0 \\ 2\left(\frac{1}{2}\right)-3y=0 \\ 1-3y=0 \\ -3y=-1 \\ y=\frac{1}{3} \end{array}$$

solution $\left(\frac{1}{2},\frac{1}{3}\right)$ check it in both original equations

Note: In above example, we had opposite coefficients of the y-terms, so when we added, the y-term was eliminated.

Sometimes we need to multiply one or both equations by a constant to eliminate a variable.

Example: solve $2x+3y=12$ ①
$5x+7y=29$ ②

Multiply both sides of ① by 5
Multiply both sides of ② by −2:

$10x+15y=60$ ①
$-10x-14y=-58$ ②

Now add ⟶ $y=2$

Now solve for x: $2x+3y=12$ ①
$2x+3(2)=12$
$2x+6=12$
$2x=6$
$x=3$

solution: $(3,2)$ check it!

Example: solve $\begin{cases} 3x+2y=2 & ① \\ 6x+4y=14 & ② \end{cases}$

multiply ① by −2: $-6x-4y=-4$ ①

Now solve : $-6x-4y=-4$ ①
$6x+4y=14$ ②
$0 = 10$ FALSE

what happened? No solutions

These are equations of parallel lines (same slope)
solution: $\varnothing$

Applications:

General Strategy for solving word problems and applications:

1. Read the problem carefully
2. Define the variable(s): Assign a variable to represent each unknown quantity. Write down what the variable represents, including units.
3. Translate the problem to an equation(s). If you have two variables, you need two equations. If you have three variables, you need three equations, etc.
4. Solve the equation(s) for each of the variables.
5. Check that your solution makes sense in the original statement of the problem.
6. Answer the question asked in the problem.

You should know how to find the cost function, revenue function, and break-even point for business appli ations

Revenue Function

$R(x) = (\text{price per unit}) * x$

Cost Function

$C(x) = \text{fixed cost} + (\text{cost per unit}) * x$

ex Your company makes calculators, fixed costs are $1,000. It costs $50 to make each calculator. You charge $80 each. Find

a. cost function
b. revenue function
c. break even point

a. $C(x) = 1000 + 50x$
b. $R(x) = 80x$
c. $1000 + 50x = 80x$

$1000 = 30x$

$33.3 = x$