Section 5.2 Systems of Equations in Three Variables

Note Title 3/9/2006

Suppose we have an equation with three variables:

$$2x + y + 2z = 5$$

This is a linear equation in three variables

The solution is an ordered triple

(x, y, z)

that makes the equation true

The graph of this equation is a plane in three-dimensional space

Suppose we have a system of 3 equations in 3 variables:

Example:
$$\begin{cases} x + y + z = 4 & (1) \\ x - 2y - z = 1 & (2) \\ 2x - y - 2z = -1 & (3) \end{cases}$$

We use a method similar to the Addition Method to solve:

ⓐ 1^{st}, we use <u>any two</u> of the equations to eliminate one of the variables and get an equation in <u>2 variables</u>:

$$x+y+z=4 \quad ①$$
$$x-2y-z=1 \quad ②$$
$$2x-y \quad =5 \quad ④$$

← get a new equation in 2 variables

ⓑ Next, use a <u>different pair</u> of equations and eliminate the <u>same variable</u>:

$$x+y+z=4 \quad ①$$
$$2x-y-2z=-1 \quad ③$$

Want to eliminate $\underline{\underline{z}}$

Mult. ① by 2 →

$$2x+2y+2z=8 \quad ①$$
$$2x-y-2z=-1 \quad ③$$
$$4x+y \quad =7 \quad ⑤$$

← another equation in 2 variables

ⓒ Now we have 2 equations in 2 variables (we know how to solve this!)

$$\begin{cases} 2x-y=5 & ④ \\ 4x+y=7 & ⑤ \end{cases}$$
$$6x=12$$
$$x=2$$

Now, solve for y using one of the equations in two variables:

$$2x-y=5 \quad ④$$
$$2(2)-y=5$$
$$4-y=5$$
$$-y=1$$
$$y=-1$$

(d) Finally, use any of the original equations to solve for z using x and y:

$x+y+z=4$ (1) ← one of the original equations

$2+(-1)+z=4$

$1+z=4$

$z=3$

Solution: $(x,y,z)=\boxed{(2,-1,3)}$

Check the solution in all three of the original equations!

ex)

$$\begin{aligned} x+y+z&=6\\ 2x\quad+z&=5\\ 2x-y+z&=3 \end{aligned}$$

the second equation doesn't have a "y" so use that as one of your 2 equations.

$$\begin{array}{r} x+y+z=6\\ 2x-y+z=3\\ \hline 3x\quad+2z=9 \end{array}$$

now solve $\begin{cases} 3x+2z=9\\ 2x+z=5 \end{cases}$

Applications

Example: On a table there are 25 coins consisting of quarters, dimes, and nickels. There are 2 more quarters than nickels. The value of all the coins is \$3.30. How many of each type of coin are there?

(a) Define the variables:

Let q = # quarters
d = # dimes
n = # dimes

(b) Translate English into Math:

"There are 25 coins"

$$q + d + n = 25 \quad (1)$$

"There are 2 more quarters than nickels"

$$q = n + 2 \quad (2)$$

"The value of the coins is \$3.30" (330 cents)

$$25q + 10d + 5n = 330 \quad (3)$$

(c) Now solve the system of equations:

$$\begin{cases} q + d + n = 25 & (1) \\ q \quad - n = 2 & (2) \leftarrow \text{rewrite with variables on left} \\ 25q + 10d + 5n = 330 & (3) \end{cases}$$

Eliminate d in (1) and (3) (since it's already eliminated from (2)):

mult. (1) by $-10 \rightarrow$

$$\begin{array}{rl} -10q - 10d - 10n = -250 & (1) \\ 25q + 10d + 5n = 330 & (3) \\ \hline 15q \quad - 5n = 80 & (4) \end{array}$$

Add (2) and (4):

mult. (2) by $-5 \longrightarrow$

$$\begin{array}{rl} -5q + 5n = -10 & (2) \\ 15q - 5n = 80 & (4) \\ \hline 10q \quad = 70 & \\ q = 7 & \end{array}$$

Solve for n: $q - n = 2$ (2) ← pick either equation in 2 variables

$7 - n = 2$

$n = 5$

Now solve for d:

$q + n + d = 25$ (1) ← pick any original equation

$7 + 5 + d = 25$

$12 + d = 25$

$d = 13$

Solution: 7 quarters, 13 dimes, 5 nickels

check it!