Section 5.5 System of Linear Inequalities

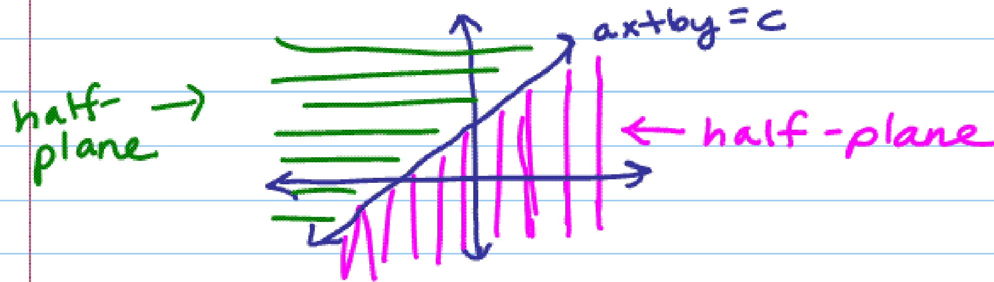
Note Title 3/25/2006

A linear inequality can be written in one of the forms:

$ax+by < c \qquad ax+by > c$

$ax+by \leq c \qquad ax+by \geq c$ where a, b, c are constants

We know the graph of a linear equality is a line in the plane:



Note that the line divides the plane into two half-planes

So really the line determines three sets:

1. The set of points below the line
2. The set of points above the line
3. The set of points on the line.

The line is said to be the boundary of the half-planes.

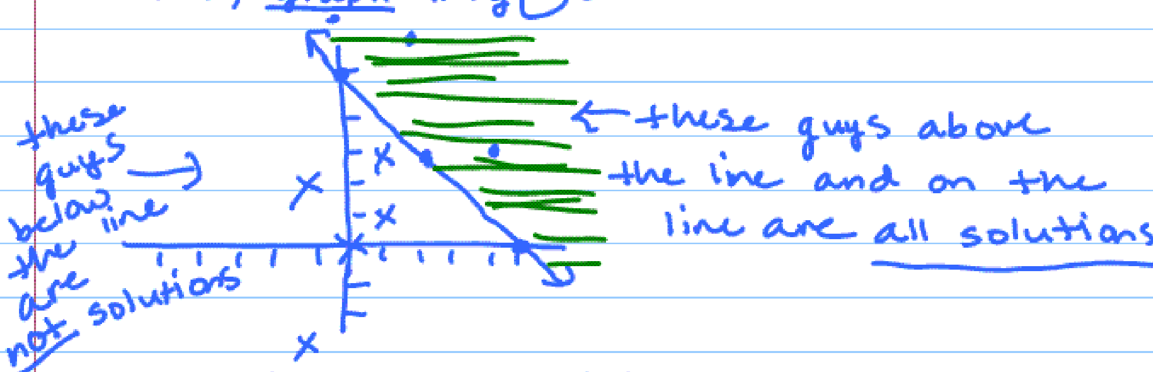
Note:

The solution to a linear inequality will be all the points in a half-plane (and maybe the boundary)

Example: $x + y \geq 5$

The solution set will be the set of all ordered pairs (x, y) that make the statement true (there will be many)

First, graph $x + y = 5$:



Now, pick some points. Are they solutions to $x + y \geq 5$?

Solutions	Not solutions
(0, 5)	(1, 1)
(2, 6)	(1, 3)
(-1, 7)	(0, 0)
(4, 3)	(-1, 2)
(2, 3)	(1, -3)

so the solution set is all points above the line and including the line (because of $\geq$). Shade the half-plane containing the solutions.

equal to part means boundary is including

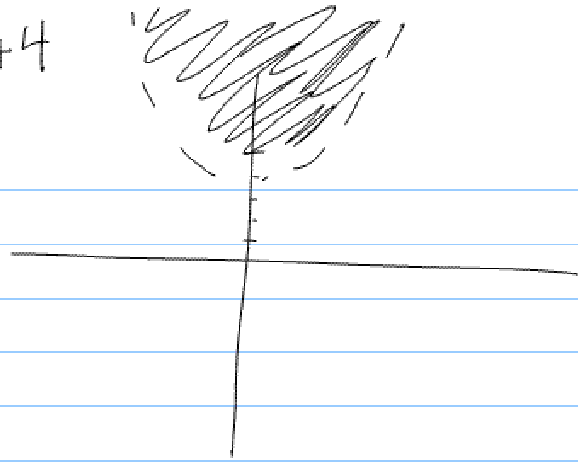
Note: since either all points in the half-plane are solutions or none of the are → we need only test one point!

To graph an inequality:

Given a linear inequality, the solution set will consist of an open ($>$, $<$) or closed ($\leq$, $\geq$) half-plane

1. Graph the associated equality (this forms the boundary of the half-plane)
2. If the inequality is $\leq$ OR $\geq$, draw a solid line
3. If the inequality is $<$ OR $>$ draw a dashed line.
4. Pick any test point not on the boundary and substitute it in the inequality
5. If the point is a solution to the inequality, shade the region containing that point.
   otherwise, shade the other region

ex) $y > x^2 + 4$



Graphing Systems of Inequalities:

Suppose you want to graph the system:

$$\begin{cases} y > -2x+10 \\ x \geq 0 \\ y \geq 0 \\ x \leq 5 \end{cases}$$

(Possible application: x is price, y is number of items. Need $y > -2x+10$ to make profit but market dictates $x \leq \$5$)

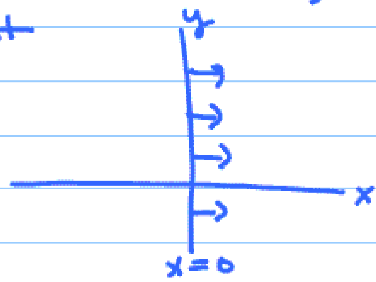
The solution to this system will consist of the intersection of the half-planes formed from the inequalities:

Example: Graph

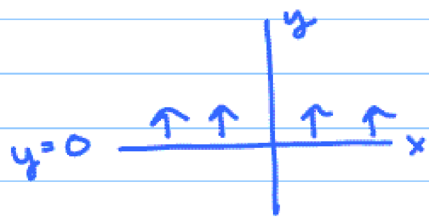
$$\begin{cases} y > -2x+10 \\ x \geq 0 \\ y \geq 0 \\ x \leq 5 \end{cases}$$

Graph each half-plane but instead of shading, make the appropriate half-plane with arrows

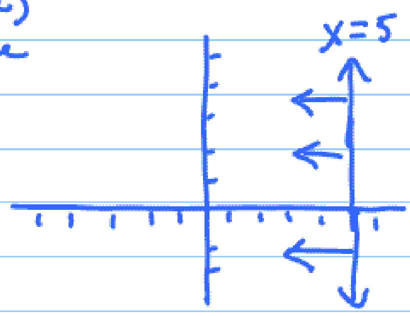
1. Graph $x \geq 0$
The vertical line $x = 0$ (y-axis) is the boundary. Any point to the right (positive direction) is in the solution set



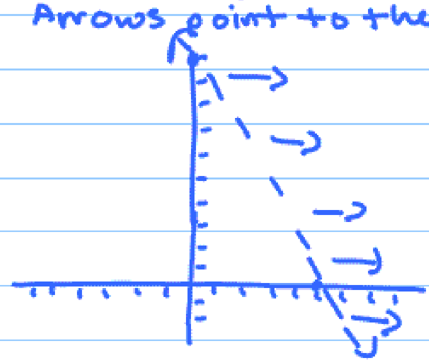
2. Graph $y \geq 0$
The horizontal line $y = 0$ (x-axis) is the boundary. Any point above the x-axis is in the solution set:



3. Graph $x \leq 5$
First graph $x = 5$ (vertical line)
Test point (0,0) : $0 \leq 5$ true
Arrows point to the left:



4. Graph $y > -2x+10$
First graph $y = -2x+10$ (dotted line)
test point $(0,0)$: $0 > 10$ False
Arrows point to the right:



5. Put it all together in one graph and shade the region forming the intersection of the regions indicated by the arrows: