Section 5.6 Linear Programming

Note Title 3/27/2006

Linear programming problems involve minimizing or maximizing some values

Examples: Maximize profits
Minimize costs

It is used in many businesses and in the military

Examples:
Schedule nurses, pizza delivery, airline routes, etc.

Usually we want to minimize or maximize a value given other limitations or constraints

Example: Diet: Minimize calories given certain nutritional requirements

Terminology: The linear equation that expresses the quantity to be minimized or maximized is called the objective function.

The constraints are expressed as a system of linear inequalities

Example ①. Suppose we want to maximize the quantity

$$C = 3x + 2y$$ ← this is the objective function

subject to the constraints:

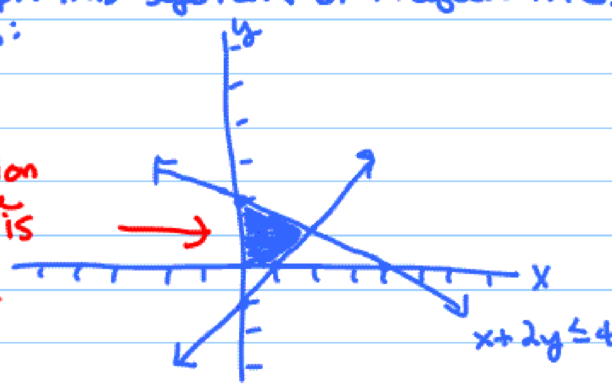
$$\begin{cases} x \ge 0 \\ y \ge 0 \\ x + 2y \le 4 \\ x - y \le 1 \end{cases}$$ ← these are the constraints

The only possible solutions are solutions to the system of constraints above.

So, graph this system of inequalities:

Graph:

the solution set to the constraints is called the feasible region



What value of (x,y) in the feasible region will maximize $C = 3x + 2y$?

It turns out: the max. and min. values of the objective function (if they exist) occur at a corner point (or vertex) of the feasible region.

Procedure for solving linear programming problem by graphing:

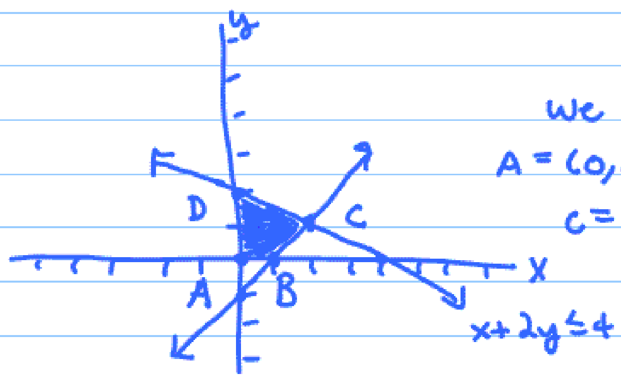
1. Define the objective function and the constraints
2. Graph the system formed from the constraints
3. Determine the corner points of the feasible region (the solution set to the system of constraints)
4. To find the max. or min., test the objective function at each of the corner points

Lets finish

Example 1:

We've already done steps 1 + 2

3. Now we want to find the corner points of the shaded region:



we have:

$A=(0,0)$, $B=(1,0)$, $C=(2,1)$, $D=(0,2)$

4. Test $C = 3x + 2y$ at each corner point

$A = (0,0)$: $C = 0 + 0 = 0$

$B = (1,0)$: $C = 3 + 0 = 3$

$C = (2,1)$: $C = 6 + 2 = 8$ ← maximum is $C = 8$

$D = (0,2)$: $C = 0 + 4 = 4$

Solution: Maximum value of $C = 8$ occurs when $x = 2$ and $y = 1$

Applications:

The most difficult part may be in determining the objective function and the constraints.

The first step should be to define the variables you are using and exactly what they represent

Example: Geek, Inc. manufactures two kinds of pocket calculators. Type A uses rechargeable batteries, type B does not. The company can make at most 50 of type A per day and 60 of type B. Type A requires 3 work-hours to produce and type B requires 2 work-hours. The work force provides a total of 180 work-hours available per day. The profit on type A is \$2.50 and the profit on type B is \$2.00. How many of each type of calculator should be produced per day to give maximum profit?

Solution:

1. Define variables:

Let a = # of type A calculators, b = # of type B calculators } unknowns

P = profit ← item to be maximized

2. It may help to make a table or organize the info. some how:

	Type A	Type B	Total
# available	50	60	
work-hours	3	2	180
Profit	\$2.50	\$2.00	P

3. Objective function:

Maximize $P = 2.50a + 2b$

4. Constraints: $a \geq 0$, $b \geq 0$ } quantities are positive

$a \leq 50$, $b \leq 60$ } make at most 50 of A and 60 of B

$3a + 2b \leq 180$ ← total work-hours available

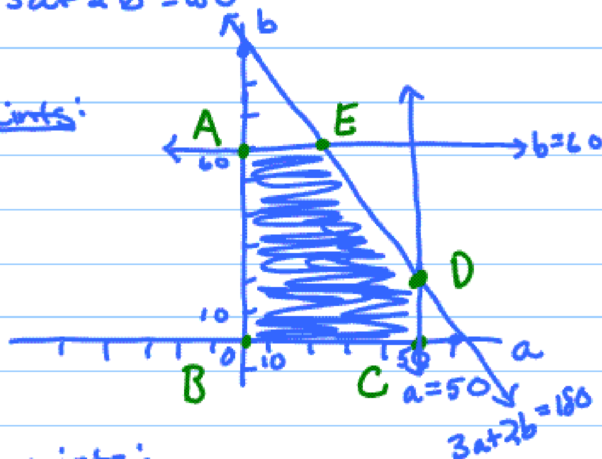
5. State the problem completely:

Maximize $P = 2.50a + 2b$

subject to $\begin{cases} a \geq 0, b \geq 0 \\ a \leq 50 \\ b \leq 60 \\ 3a + 2b \leq 180 \end{cases}$

6. Solve:

Graph constraints:



Find corner points:

A = (0,60), B = (0,0), C = (50,0),

D: solve $\begin{cases} a = 50 \\ 3a + 2b = 180 \end{cases}$ substitute ⇒ $b = 15$

D = (50,15)

E: solve $\begin{cases} b = 60 \\ 3a + 2b = 180 \end{cases}$ substitute ⇒ $a = 20$

E = (20,60)

Test corner points:

corner point	$P = 2.5a + 2b$ (maximize)
(0,60)	P = 120
(0,0)	P = 0
(50,0)	P = 125
(50,15)	P = 155
(20,60)	P = 170 maximum

7. check that the solution make sense in the original problem.

8. state the complete solution:

The maximum profit of \$170 per day occurs when the company makes 20 type A and 60 type B calculators.

Phew!