Section 8.1 Sequences and Summation Notation

Note Title 4/1/2006

Sequences

A sequence is a collection of objects listed in an ordered way (pattern).

Examples:

(a) $1, 3, 5, 7, \ldots$ ← infinite sequences
(b) $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots$

Notation and terminology:

we could talk about example (b) above as follows:

term number	terms (ex (b))	terms (general)
1	$1/2$	a_1
2	$1/4$	a_2
3	$1/8$	a_3
4	$1/16$	a_4
$\vdots$		$\vdots$
n	$1/2^n$	a_n
$\vdots$	$\vdots$	$\vdots$

We call the n^{th} term or general term of the sequence, a_n
We denote the entire sequence by $\{a_n\}$

It is often helpful to find a rule for finding the n^{th} term of a sequence (then we can find any term of a sequence without having to find all the ones before it)

Example (b) again:

We found $a_n = \frac{1}{2^n}$

Now we can use that rule to find any term of the sequence:

6th term: $a_6 = \frac{1}{2^6} = \frac{1}{64}$

11th term: $a_{11} = \frac{1}{2^{11}} = \frac{1}{2048}$

Another example:

write the first four terms of the sequence given by the rule

$$a_n = 1 + 2(n-1)$$

Solution: $a_1 = 1 + 2(1-1) = 1$

$a_2 = 1 + 2(2-1) = 3$

$a_3 = 1 + 2(3-1) = 5$

$a_4 = 1 + 2(4-1) = 7$

This is the sequence $1, 3, 5, 7, \ldots$

Hey! this was example (a) above!

Fibonacci Sequence

The Fibonacci Sequence is an example of a sequence discovered long ago by an Italian named Fibonacci.

It's a sequence of numbers that are found frequently in nature and in music, art, architecture and other areas that rely on "pleasing" forms

Fibonacci sequence:

1, 1, 2, 3, 5, 8, 13, 21, 34, ...

Can you see the pattern?

Each term is the sum of the two preceding terms.

For more cool information about Fibonacci sequences, go to:

www.textism.com/bucket/fib.html

OR

www.ualr.edu/lasmoller.fibonacci.html

Factorials

In sequences and other applications we often have expressions like:

$$7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$$

It would be nice to have some notation for these expressions:

If n is a positive integer, then
n factorial is defined by:

$$n! = n \cdot (n-1) \cdot (n-2) \cdots 3 \cdot 2 \cdot 1$$

We also define $0! = 1$

Examples: Find the following

a) $0! = 1$
b) $1! = 1$
c) $2! = 2 \cdot 1 = 2$
d) $3! = 3 \cdot 2 \cdot 1 = 6$
e) $4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$
f) $12! = 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdots 3 \cdot 2 \cdot 1 = 479{,}001{,}600$

Examples: Evaluate the following

a) $\dfrac{9!}{7!} = \dfrac{9 \cdot 8 \cdot \cancel{7} \cdot \cancel{6} \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}{\cancel{7} \cdot \cancel{6} \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}} = 72$

Short cut: $\dfrac{9!}{7!} = \dfrac{9 \cdot 8 \cdot \cancel{7!}}{\cancel{7!}} = 72$

b) $\dfrac{n!}{(n-1)!} = \dfrac{n \cdot \cancel{(n-1)!}}{\cancel{(n-1)!}} = n$

Factorials are useful in defining sequences:

Example: write the first three terms of the sequence defined by

$$a_n = \frac{(n+1)!}{n^2}$$

Solution: $a_1 = \frac{(1+1)!}{1^2} = \frac{2!}{1} = \frac{2\cdot 1}{1} = 2$

$a_2 = \frac{(2+1)!}{2^2} = \frac{3!}{4} = \frac{3\cdot 2\cdot 1}{4} = \frac{3}{2}$

You try it! $a_3 = \frac{(3+1)!}{3^2} = \frac{4!}{9} = \frac{4\cdot 3\cdot 2\cdot 1}{9} = \frac{24}{9} = \frac{8}{3}$

Summation Notation

Sometimes we want to find the sum of the first n terms of a sequence:

$$a_1 + a_2 + a_3 + \ldots + a_n$$

shorthand notation:

this is the Greek letter sigma (for sum) $\rightarrow$ $\sum_{i=1}^{n} a_i = a_1 + a_2 + a_3 + \ldots + a_n$

$\leftarrow$ i is the counter for each term

Examples:

(a) $\sum_{i=1}^{4} i^2 = 1^2 + 2^2 + 3^2 + 4^2$
$= 1 + 4 + 9 + 16 = 30$

(b) $\sum_{j=0}^{4} (j-5) = (0-5) + (1-5) + (2-5) + (3-5) + (4-5)$
$= -5 + -4 + -3 + -2 + -1$
$= -15$

(c) $\sum_{k=1}^{4} 7 = 7 + 7 + 7 + 7 = 28$

Example: Write in summation notation:

(a) $5+5^2+5^3+\ldots+5^{12}$ ← twelve terms

$= \sum_{i=1}^{12} 5^i$

(b) $\frac{1}{9}+\frac{2}{9^2}+\frac{3}{9^3}+\ldots+\frac{n}{9^n}$ ← n terms

$= \sum_{i=1}^{n} \frac{i}{9^i}$

(c) You try this one!

$a+ar+ar^2+\ldots+ar^{14}$ ← 15 terms

$\sum_{i=1}^{15} ar^{(i-1)}$ $\left(\text{OR } \sum_{i=0}^{14} ar^i\right)$