8.2 Arithmetic Sequences

Note Title 4/3/2006

An **arithmetic sequence** is a sequence in which consecutive terms have a **common difference**

(In other words to get the next term, you add a constant amount to the preceding term)

Example ①:

arith. sequence: 1, 5, 9, 13, ...

$d=4$ $d=4$ $d=4$

difference, d, between terms is 4

The **common difference** is $d=4$

General term

Find a formula for **general term**, a_n, of an arithmetic sequence with common difference d:

term #	term
1	a_1
2	a_1+d
3	$a_1+d+d = a_1+2d$
4	$a_1+2d+d = a_1+3d$
5	$a_1+3d+d = a_1+4d$
⋮	⋮

In general, we have

* $\boxed{a_n = a_1 + (n-1)d}$ General term of arithmetic sequence

Example: Find the general term for example ① above and use it to find the 10th term:

Sequence: 1, 5, 9, 13, ...

General term: $a_n = a_1 + (n-1)d$

with $a_1 = 1$, $d = 4$

$a_n = 1 + (n-1)4$

Find 10th term:

$a_{10} = 1 + (10-1)4$

$= 1 + 36 = 37$

See examples 2 and 3 in this section of your text

Sum of n terms of Arithmetic Sequence

The nth partial sum of an arithmetic sequence is given by:

$S_n = a_1 + a_2 + \ldots + a_n$

Isn't there an easier way to find this without having to add up all the terms?

A messy and tricky way to find a better formula:

$$S_n = a_1 + a_2 + a_3 + \ldots + a_n \Rightarrow$$

add → $S_n = a_1 + (a_1+d) + (a_1+2d) + \ldots + a_n$

So $S_n = a_n + (a_n-d) + (a_n-2d) + \ldots + a_1$

trick: write terms backwards

$$2S_n = \underbrace{(a_1+a_n) + (a_1+a_n) + (a_1+a_n) + \ldots + (a_1+a_n)}_{n \text{ terms}}$$

So, $2S_n = n(a_1+a_n)$

$$\boxed{S_n = \frac{n}{2}(a_1+a_n)}$$ (n^{th} partial sum of arithmetic sequence)

Example: find the sum of the first 30 terms of the arithmetic sequence

1, 5, 9, 13, 17, ...

We have $n = 30$, or

$$S_{30} = \frac{30}{2}(a_1 + a_n)$$

We know $a_1 = 1$, we need to know the 30th term, a_{30}

Use formula for general term:

$$a_n = a_1 + (n-1)d, \quad d = 4$$

$$a_{30} = 1 + (30-1)4 = 1 + 29 \cdot 4 = 117$$

So, $S_{30} = \frac{30}{2}(1 + 117) = 15(118)$

$$= 1770$$