Section 8.3 Geometric Sequences

Note Title 4/3/2006

A geometric sequence is a sequence in which consecutive terms have a <u>common ratio</u>, r.

(In other words, to get next term, <u>multiply</u> preceding term by a nonzero constant)

<u>Examples</u>: (a) $3, 6, 12, 24, \ldots$

times 2 times 2

common ratio, $r = 2$

(b) $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots$

times $\frac{1}{2}$

$r = \frac{1}{2}$

(c) $1, -3, 9, -27, 81, \ldots$

$r = -3$

<u>General term</u>

Find formula for general term, a_n

General geometric sequence:

Let r = common ratio

term #	term
1	a_1
2	$a_1 r$
3	$a_1 r^2$
4	$a_1 r^3$
5	$a_1 r^4$
$\vdots$	$\vdots$

so, we have general term:

$\boxed{a_n = a_1 r^{n-1}}$ General term of a geometric sequence

Example: Find the 12th term of the geometric sequence with 1st term = 3 and common ratio = −2

$a_1 = 3, \; r = -2$

$a_n = a_1 r^{n-1}$

$a_{12} = 3(-2)^{11} = 3(-2048) = -6144$

Sum of first n terms of geom. sequence

The nth partial sum, S_n, of a geometric sequence is given by:

$S_n = a_1 + a_2 + a_3 + \dots + a_n$

Find a better formula:

$S_n = a_1 + \cancel{a_1 r} + \cancel{a_1 r^2} + \dots + \cancel{a_1 r^{n-2}} + \cancel{a_1 r^{n-1}}$

$r S_n = \cancel{a_1 r} + \cancel{a_1 r^2} + \cancel{a_1 r^3} + \dots + \cancel{a_1 r^{n-1}} + a_1 r^n$

trick: multiply both sides by r

$S_n - r S_n = a_1 - a_1 r^n$

$S_n (1 - r) = a_1 (1 - r^n)$

$\boxed{S_n = \dfrac{a_1 (1 - r^n)}{1 - r}}$ nth partial sum of geom. seq.

Example: Find the sum of the first 12 terms of the geometric sequence:

3, 6, 12, 24, ...

we have $a_1 = 3$, $r = 2$

$$S_{12} = \frac{3(1-2^{12})}{1-2} = \frac{3(-4095)}{-1}$$

$= 12{,}285$

Note: Geometric sequences are really exponential functions and get big very quickly!

Summary of Sections 8.2 and 8.3:

For an arithmetic sequence (consecutive terms have common difference, d)

1. General term: $a_n = a_1 + (n-1)d$
2. n^{th} partial sum: $S_n = \frac{n}{2}(a_1 + a_n)$

For a geometric sequence (common ratio, r)

1. General term: $a_n = a_1 r^{n-1}$
2. n^{th} partial sum: $S_n = \frac{a_1(1-r^n)}{1-r}$