Section 8.6 Counting Principles

Note Title 4/15/2006

The first step in calculating probability is to figure out how many ways there are to do something.

Example: Powerball – to determine probability of winning you first need to figure out how many different combinations of numbers there are.

So, first we need to learn ways of counting things: combinatorics

Example: A fast food restaurant offers a special: choice of 2 soups (noodle, tomato); choice of 3 sandwiches (burger, chicken, tofu); choice of 2 drinks (coke, 7up) for $4.99. How many different meal combinations are there?

(a) Method 1: List them all

1. noodle soup
   burger
   coke

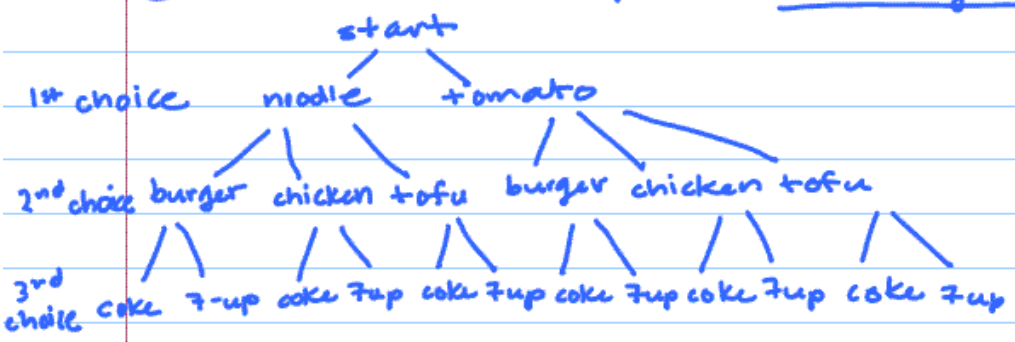
2. noodle soup
   chicken
   coke

3. noodle soup
   tofu
   coke

4. tomato soup
   burger
   coke

..... etc.

(b) Method 2: Draw a picture - Tree Diagram



number of final branches is number of possibilities = 12.

Each time we had a decision, the number of branches was multiplied by the number of choices for that decision

we had,

$2 \cdot 3 \cdot 2 = 12$ combinations

2 soups · 3 sandwiches · 2 drinks

* **Fundamental Counting Principle**

The number of ways in which a series of decisions can occur is found by multiplying the number of ways each decision can occur

Example: An art collector has 2 paintings by Picasso, 4 by Matisse, and 3 by Van Gogh. How many ways can she hang one by each artist?

3 decisions: $\underset{\text{Picasso}}{2} \cdot \underset{\text{Matisse}}{4} \cdot \underset{\text{Van Gogh}}{3} = 24$

Permutations: Number of ways to select things when **order** of selection matters

Example: 200 people buy raffle tickets. How many ways are there of awarding 1st, 2nd, 3rd prizes?

We want to choose 3 tickets out of 200, **order** matters. From Fund. Counting Principle, we have

3 decisions: $\underset{\text{1st place}}{200} \cdot \underset{\text{2nd place}}{199} \cdot \underset{\text{3rd place}}{198} = 7,880,400$

How about a formula for what we just did?

The **number of permutations** of r items from a set of n items is

$$ {}_nP_r = \frac{n!}{(n-r)!} $$

Above example:

Pick 3 items of out a set of 200 and order matters. This is a permutation of $r=3$ out of a set of $n = 200$:

$$P_{200\ 3} = \frac{200!}{(200-3)!} = \frac{200!}{197!}$$

$$= \frac{200 \cdot 199 \cdot 198 \cdot \cancel{197!}}{\cancel{197!}}$$

$$= 7{,}880{,}400$$

Combinations: Number of ways to select things but order of selection is not important.

Example: 200 people buy raffle tickets. You choose 3 for the top prize (same prize). How many ways are there of doing this?

We want to choose 3 tickets out of 200, order doesn't matter.

The number of combinations of r items from a set of n items is

$$_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

Back to example above:

We have

$$_{200}C_3 = \frac{200!}{3!\,197!} = \frac{200 \cdot 199 \cdot 198}{3 \cdot 2} = 1{,}313{,}400$$

Another example:

Suppose out of a class of 25 students, we want 2 students to go out and buy us all doughnuts. How many ways could those 2 be chosen?

Order doesn't matter $\Rightarrow$ combinations

$$_{25}C_2 = \frac{25!}{2!\,23!} = \frac{25 \cdot 24 \cdot 23!}{2!\,23!} = \frac{600}{2} = 300$$

Powerball

Find how many possible numbers there are:

Have to match 5 numbers from 1 to 53 and then one number from 1 to 42

O O O O O | O

white balls (order doesn't matter) | red ball

$$_{53}C_5 \cdot 42$$ ← from Fund. Counting Princ.

$$_{53}C_5 \cdot 42 = \frac{53!}{5!\,48!} \cdot 42$$

$$= \frac{53 \cdot 52 \cdot 51 \cdot 50 \cdot 49}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} \cdot 42$$

$$= \frac{14,463,212,400}{120}$$

$$= 120,526,770$$

(Prob. winning number $\approx \frac{1}{120 \text{ million}}$)