Section 8.7 Probability

Note Title 4/17/2006

(A little background terminology and notation first:

Sets

we know a set is a collection of objects:

Examples: $A = \{1, 2, 3\}$
$S = \{A, B, C, D\}$

Each object in the set is called an element

The cardinal number of a set A is the number of elements in the set - denoted $n(A)$

Examples: above sets
$n(A) = 3$
$n(S) = 4$

Note: $n(\emptyset) = 0$)

Probability

Probability - a number that indicates the likelihood that something will happen - will always be between 0 and 1 (closer to 1 $\Rightarrow$ more likely)

Terminology

1. An experiment is the observation of an occurrence (or outcome)
2. The set, S, of all possible outcomes of an experiment is called the sample space
3. A set of possible outcomes of an experiment is called an event, E.

Example:

Experiment: Toss a coin

Outcomes: H = "heads", T = "tails"

Sample space: $S = \{H, T\}$

Event: Getting "heads", $E = \{H\}$

In general, if an experiment has equally likely outcomes, the given an event E,

$$p(E) = \frac{\text{number of ways that } E \text{ can occur}}{\text{number of all possible outcomes}} = \frac{n(E)}{n(S)}$$

Example: Experiment: Toss a six-sided die

$S = \{1, 2, 3, 4, 5, 6\}$

a) Find probability of getting a 5

$E = \{5\}$

$p(E) = \frac{n(E)}{n(S)} = \frac{1}{6}$

(b) Find probability of getting a number greater than 3

$E = \{4, 5, 6\}$

$p(E) = \frac{n(E)}{n(S)} = \frac{3}{6} = \frac{1}{2} = .50 = 50\%$

Another example: Two coins are tossed.

Find:

(a) Probability of getting heads on 1st coin

all possibilities $\Rightarrow S = \{HH, HT, TH, TT\}$

getting heads on 1st coin $\Rightarrow E = \{HH, HT\}$

$p(E) = \frac{n(E)}{n(S)} = \frac{2}{4} = \frac{1}{2} = 50\%$

(b) Probability of getting heads on at least one coin

$E = \{HH, HT, TH\}$

$p(E) = \frac{n(E)}{n(S)} = \frac{3}{4} = .75 = 75\%$

Often you don't have to list all the possible outcomes in the sample space, S. You just have to know how many there are $\Rightarrow$ use combinatorics

Example: A die is rolled, then two coins are tossed. Find the probability of rolling an even number, then getting two heads.

$E = \{(2,HH), (4,HH), (6,HH)\}$

How many possible outcomes are there?
From Fundamental Counting Principle, we know:

$$n(S) = \underset{\substack{\text{number}\\\text{of different}\\\text{rolls of die}\\(1,2,3,4,5,6)}}{\underline{6}} \cdot \underset{\substack{\text{number}\\\text{of coin}\\\text{outcomes}\\\text{(heads or}\\\text{tails)}}}{\underline{2}} \cdot \underset{\substack{\text{number}\\\text{of coin}\\\text{outcomes}}}{\underline{2}} = 24$$

So, $p(E) = \frac{n(E)}{n(S)} = \frac{3}{24} = \frac{1}{8}$

Powerball

In the last section (section 8.6), we saw that the number of all possible combinations of Powerball tickets was:

$_{53}C_5 \cdot 42$

So, if E is the event of winning Powerball, then

$$p(E) = \frac{n(E)}{n(S)} = \frac{\text{number of winning combinations}}{\text{number of all possible combinations}}$$

$$= \frac{1}{_{53}C_5 \cdot 42}$$

$$= \frac{1}{120{,}526{,}770} \approx \frac{1}{120\text{ million}}$$

Wow!