1.1

Fractions:

ex $\frac{3}{4}$ ← numerator / ← denominator

natural numbers:

1, 2, 3, 4, 5, 6, . . .

Mixed numbers:

ex $2\frac{1}{4}$

Improper fraction: has a numerator that is bigger than denominator

ex $\frac{7}{2}$

Converting a mixed # to an improper fraction

Technically, a mixed # is a natural # plus a fraction.

ex $2\frac{1}{4} = 2 + \frac{1}{4}$

$= \frac{8}{4} + \frac{1}{4}$ ← rewrite 2 as $\frac{8}{4}$ to have common denominators.

$= \boxed{\frac{9}{4}}$ ← improper fraction!

Shortcut,

$$A\frac{b}{c} = \frac{c \cdot A + b}{c}$$

ex $2\frac{1}{4} = \frac{4 \cdot 2 + 1}{4}$

$= \frac{8+1}{4}$

$= \boxed{\frac{9}{4}}$

Converting Improper fractions to Mixed

Step 1: take numerator divided by denominator until you get a remainder (no decimals)

Step 2: Mixed number is

answer $\frac{\text{remainder}}{\text{original denominator}}$

ex $7/2$

$$2\overline{)7} = 3, \quad 7 - 6 = 1 \leftarrow \text{remainder!}$$

$3\frac{1}{2}$

Using Variables in fractions

In algebra we use letters to represent numbers. These letters are called variables

ex $\frac{a}{b}$

Sometimes we choose values for these variables

ex If we choose $a=4$ and $b=6$ then

$a/b = 4/6$

Factors, Prime & Composite #s

ex $2 \cdot 3 = 6 \leftarrow$ product - the answer when multiplying

$\uparrow$ factors - the #'s being multiplied

We say 2 and 3 are factors of 6

Some #s have lots of factors!

ex the factors of 24 are 1, 2, 3, 4, 6, 8, 12, 24

Since $1 \cdot 24 = 24$ $\quad 2 \cdot 12 = 24$ $\quad 3 \cdot 8 = 24$ $\quad 4 \cdot 6 = 24$

Some #s only have two factors

ex the factors of 5 are 1 and 5

· since $1 \cdot 5 = 5$

Numbers such as 5 are called <u>prime #s</u>

<u>prime number</u> - a # whose only factors are 1 and itself (nothing else!)

<u>composite number</u> - a # that is NOT prime (aka it has more than two factors.)

ex. 20 is composite because $4 \cdot 5 = 20$
(and $2 \cdot 10 = 20$ and $1 \cdot 20 = 20$)

7 is prime because $1 \cdot 7 = 7$ but nothing else!

*Notice: when finding factors we only use the natural #s (that is no negatives, no decimals/fractions)

<u>Prime factorization</u>

Writing a composite # as a bunch of prime numbers multiplied together.

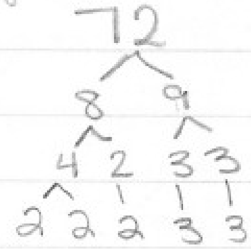
ex

12
2 6
2 2 3 ← all prime #s

so the prime factorization of 12 is $2 \cdot 2 \cdot 3$
(notice: $2 \cdot 2 \cdot 3 = 4 \cdot 3 = 12$ ✓)

bigger the # the harder this is

(ex)



p.f. of 72 is $2\cdot2\cdot2\cdot3\cdot3$

<u>Reducing fractions</u>

Fundamental Principle of fractions: If a # is <u>multiplied</u> in the numerator & the denominator they "cancel"

in symbols: $\frac{a\cdot c}{b\cdot c} = \frac{a}{b}$

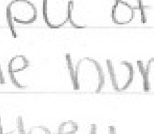
(ex) $\frac{6}{8} = \frac{\cancel{2}\cdot3}{\cancel{2}\cdot4} = 3/4$

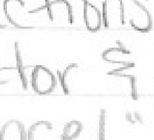
To Completely Reduce a fraction

1. Find prime factorization of numerator
   " " denominator

2. Cancel whatever cancels.

(ex.) $\frac{16}{24}$





$\frac{16}{24} = \frac{\cancel{2}\cdot\cancel{2}\cdot\cancel{2}\cdot2}{\cancel{2}\cdot\cancel{2}\cdot\cancel{2}\cdot3} = \frac{2}{3}$

Note: this "cancelling" only works with MULTIPLICATION!

(non-ex) $\frac{2+3}{2+4}$ ← the 2's do NOT cancel!

Multiplying fractions

to multiply fractions
- multiply the numerators
- " " denominators

$\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}$

ex $\frac{2}{3} \cdot \frac{5}{6} = \frac{2 \cdot 5}{3 \cdot 6}$

$= \boxed{10/18}$ reduce → $\frac{\cancel{2} \cdot 5}{\cancel{2} \cdot 3 \cdot 3} = \boxed{5/9}$

To divide fractions
- change the division to multiplication
- flip the 2nd fraction over (take the reciprocal)

$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$

ex $\frac{2}{9} \div \frac{1}{4} = \frac{2}{9} \cdot \frac{4}{1}$

$= \boxed{8/9}$ reduce → $\frac{2 \cdot 2 \cdot 2}{3 \cdot 3}$ nothing cancels → $\boxed{8/9}$

Some tricky things!

#1. all integers can be written as fractions w/ a 1 for a denominator ex $5 = \frac{5}{1}$

#2. all mixed #s can be written as improper fractions ex $3\,2/5 = 17/5$

ex $\frac{6}{7} \cdot 4\frac{1}{2} = \frac{6}{7} \cdot \frac{9}{2} = \boxed{\frac{54}{14}}$ reduce → $\boxed{\frac{27}{7}}$

$\frac{4}{3} \div 2 = \frac{4}{3} \div \frac{2}{1}$

$= \frac{4}{3} \cdot \frac{1}{2} = \boxed{\frac{4}{6}}$ reduce → $\boxed{\frac{2}{3}}$

To add/subtract fractions

if the denominators are the same

- add/subtract numerators
- keep same denominator

if the denominators are different

- make denominators the same (get common den.)
- do above stuff

(ex) $\underbrace{\frac{3}{5} + \frac{1}{5}}_{\text{same}} = \frac{3+1}{5} = \boxed{\frac{4}{5}}$ $\qquad \frac{6}{9} - \frac{4}{9} = \frac{6-4}{9} = \boxed{\frac{2}{9}}$

(ex) $\frac{1}{2} + \frac{3}{4}$

here the den. are different, so we want to make them the same (or get common denominators)

To get common denominators we want to "undo" some cancelling

(ex) notice that $\frac{2}{4}$ when reduced is $\frac{1\cdot 2}{2\cdot 2} = \frac{1}{2}$

so $\frac{2}{4} \to \frac{1}{2}$, now we want to "put the two's back in"

$$\frac{1}{2} = \frac{1\cdot 2}{2\cdot 2} = \frac{2}{4} \text{ so } \frac{1}{2} \to \frac{2}{4}$$

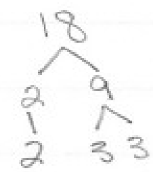
(ex) from above $\frac{1}{2} + \frac{3}{4} = \frac{2}{4} + \frac{3}{4} = \frac{2+3}{4} = \frac{5}{4}$

But how did I know to multiply the $\frac{1}{2}$ by $\frac{2}{2}$?

Two ways
1. Using Prime factorization
2. Using denominator shortcut

Prime factorization Method
- find prime factorization of both den.
- multiply each fraction by whichever factors it's missing. (on top & bottom)

ex. $\frac{5}{18} + \frac{2}{21}$



$\frac{5}{2\cdot3\cdot3} + \frac{2}{7\cdot3}$

P.f. of $18 = 2\cdot3\cdot3$ P.f. of $21 = 7\cdot3$

$\frac{5\cdot7}{2\cdot3\cdot3\cdot7} + \frac{2\cdot2\cdot3}{7\cdot3\cdot2\cdot3}$

$\frac{5\cdot7}{18\cdot7} + \frac{2\cdot2\cdot3}{21\cdot2\cdot3} = \frac{35}{126} + \frac{12}{126} = \boxed{\frac{47}{126}}$

Denominator Shortcut
- multiply 1st fraction (on top & bottom) by den. of 2nd fraction
- multiply 2nd fraction (on top & bottom) by den. of 1st fraction.

ex. $\frac{5}{18} + \frac{2}{21} = \frac{5\cdot21}{18\cdot21} + \frac{2\cdot18}{21\cdot18}$

$= \frac{105}{378} + \frac{36}{378} = \boxed{\frac{141}{378}}$ reduce → $\frac{47\cdot3}{126\cdot3} = \boxed{\frac{47}{126}}$