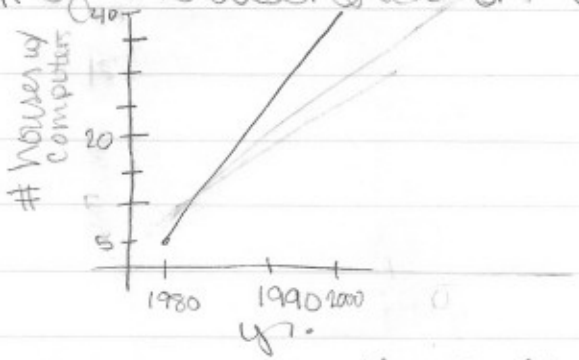
4.3

Rates of Change

(ex) # of households in town w/ a computer.



you can see that the # of computers increased but how fast did it increase? This is the "rate of change"

$$\text{rate of change} = \frac{\text{change in \# of computers}}{\text{change in yrs.}}$$

$$= \frac{40-5}{2000-1980} = \frac{35}{20} = 1.75$$

so the # of computers increased about 1.75 comp./yr.

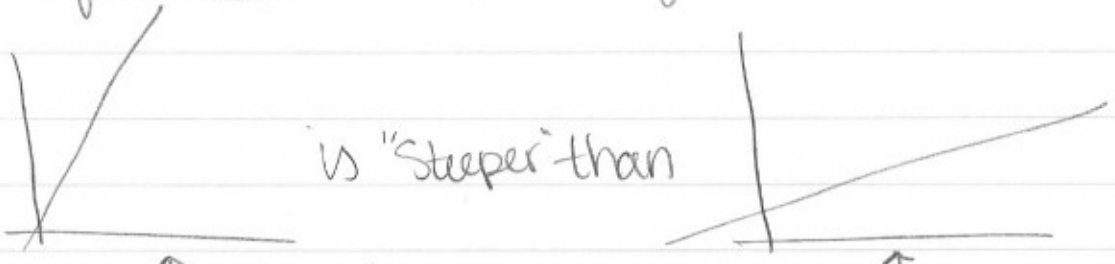
Slope of a Line

In my previous ex. I found the rate of change of houses w/ computers. But notice that the line is increasing at the same rate.

We call this the slope

Slope of a line - "steepness" of a line.

ex)

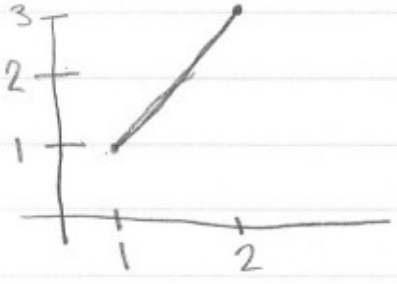


is "steeper" than

so ↑ slope is greater than →

Now I don't have $\frac{\text{change in \# comp.}}{\text{change in yrs.}}$, now I have $\frac{\text{change in y}}{\text{change in x}}$.

ex)



$\frac{\text{change in y} = 2}{\text{change in x} = 1}$ so slope $= \frac{2}{1} = 2$.

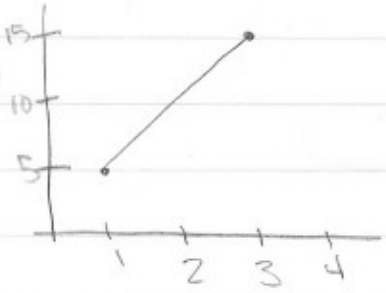
We call this $\frac{\text{rise}}{\text{run}}$

Formula for Slope

The slope, m, between two points, (x_1, y_1) & (x_2, y_2), is

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

ex



$(x_1, y_1) = (1, 5)$
$(x_2, y_2) = (3, 15)$

$$m = \frac{15-5}{3-1} = \frac{10}{2} = \boxed{5}$$

Be CAREFUL! You can pick either point first but you need to keep the xs & ys together.

ex from above $x_2 = 1$ and $y_2 = 5$ is OK

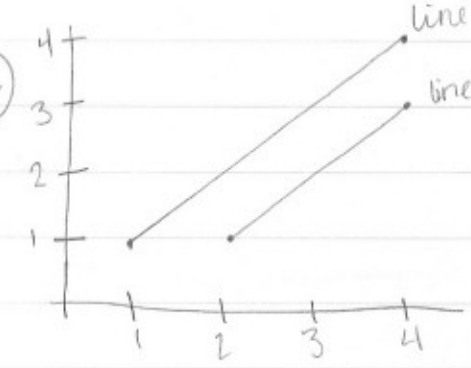
but CANNOT choose ~~$x_1 = 1$ and $y_2 = 15$~~.

BE CAREFUL! Be sure to subtract in the right order.

$\frac{y_2 - y_1}{x_2 - x_1}$ is OK but DON'T DO $\frac{y_2 - y_1}{x_1 - x_2}$.

Two lines with the same slope are parallel (because they have the same "steepness")

ex.



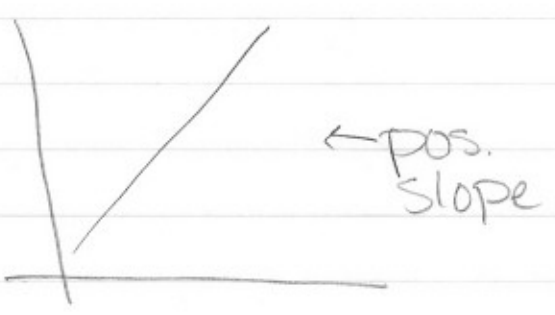
$$m_1 = \frac{4-1}{4-1} = \frac{3}{3} = 1$$

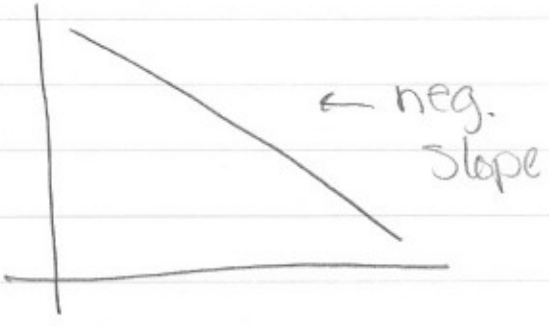
$$m_2 = \frac{3-1}{4-2} = \frac{2}{2} = 1$$

Positive, Negative, Zero, and Undefined Slopes

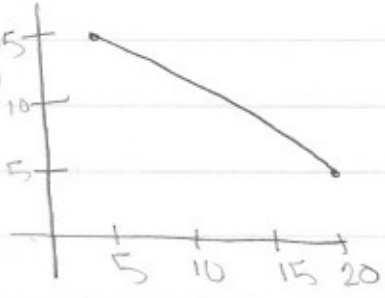
When "reading" the graph from left to right,
if the line goes up, the slope is positive,
if the line goes down, the slope is negative.

ex





ex



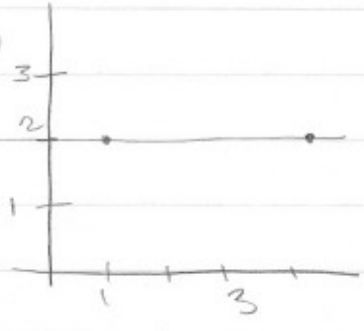
$(x_1, y_1) = (5, 15)$
$(x_2, y_2) = (20, 5)$

$m = \frac{5-15}{20-5} = \frac{-10}{15} = -\frac{2}{3}$

or $-.\overline{66}$

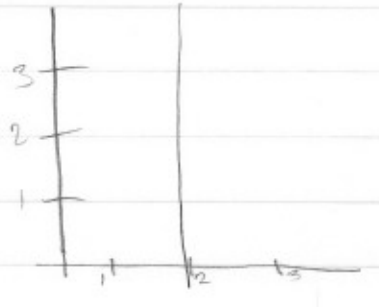
A horizontal line has a slope of 0.
A vertical line has an undefined slope.

ex



$(x_1, y_1) = (1, 2)$
$(x_2, y_2) = (4, 2)$

$m = \frac{2-2}{4-1} = \frac{0}{3} = 0$



$(x_1, y_1) = (2, 1)$
$(x_2, y_2) = (2, 2)$

$m = \frac{2-1}{2-2} = \frac{1}{0}$ undefined