

6.1 Polynomials

Recall, a term is a number, variable(s) or a product of number(s) & variable(s).

ex) $3x^4$, $3x^4 + x^3$, $\frac{1}{4}x^2 + \frac{1}{3}x + \frac{1}{5}$

\uparrow \uparrow \uparrow \uparrow \uparrow
 coefficient term term terms terms

Polynomial - a sum of terms in which all variables have whole number exponents.

idea: have ^{all} terms with coefficients and all variable exponents are pos. ints.

ex) $4x^2 + 3x - 5$
 $\frac{1}{10}x^4 - 9x$
 $\sqrt{2}x^9 + 2x^7 - 3^{-4}x^6$
 $8x^3 + 7xy^2 - 4y$

} polynomials

non-ex)

$4x^{-2} + 3x - 5$
 $\frac{x+4}{x^2}$
 $x^3 - 2x^2 + 2\sqrt{x} - 3$

} not polynomials

We write polynomials in decreasing powers.

ex) $4x + 3x^4 - 2 \rightarrow 3x^4 + 4x - 2$

Monomials, Binomials, and Trinomials

monomial - polynomial with one term	ex. $3x^4, 4x^2, 5x, 6$
binomial - " 2 terms	ex. $3x^4 + 3x, x^5 + 2$
trinomial - " 3 terms	ex. $4x^2 + 2x + 1, x^6 + x^2 + 3x$

ex) $\frac{1}{3}x^{50} - \frac{1}{2}x^{49}$ ← binomial
 $5x^2$ ← monomial
 $7x^{100} - 6x + 3$ ← trinomial

Degree of a Polynomial

The degree of a polynomial is the largest power in the polynomial.

⊗ degree of $7x^4 - 3x^2 + x - 4$ is 4 since x^4 is the largest power!

⊗ degree of $x^2 + 4x^4 + 2x^{10} - x$ is 10.

leading term - the first term when written in descending order.

leading coefficient - coefficient of leading term

⊗ $4x^2 - 5x^4 + 2x - 3 \rightarrow -5x^4 + 4x^2 + 2x - 3$

So leading term = $-5x^4$

leading coefficient = -5 .

Adding Monomials

Recall, like terms have same variables with the same exponents.

ex) $3x$ and $-4.26x$
 $-1.5x^4$ and $4x^4$
 $\frac{1}{3}x^2y^3$ and $2x^2y^3$ } like terms

non-ex)
 $3x$ and $7y$
 $5x^2$ and $2x$
 $\frac{2}{5}xy$ and $\frac{2}{5}xy^2$ } unlike terms

Recall, when adding terms:
 like terms - combine coefficients
 unlike terms - stuck as they are

ex) $3x - 4.26x = -1.26x$
 $3x + 7y \leftarrow$ stuck

To add monomials, combine like terms

ex) Add $6x^4$ and $9x^4 \rightarrow 6x^4 + 9x^4 = (6+9)x^4 = 15x^4$
 Add $3x^2y$ and $2xy \rightarrow 3x^2y + 2xy$
 Add $4xy$ and $-3xy \rightarrow 4xy + -3xy = (4-3)xy = 1xy = xy$

Subtracting Monomials

Combine like terms.

$$\textcircled{\text{ex}} \quad 12x^3 - 8x^3 = (12-8)x^3 = 4x^3$$

Subtract $3x^2y^4$ from $6x^2y^4$

$$6x^2y^4 - 3x^2y^4 = (6-3)x^2y^4 = 3x^2y^4$$

$$\textcircled{\text{ex}} \quad 3x^3 - 2x^2 \text{ just that}$$

$$\textcircled{\text{ex}} \quad \text{subtract } 15xy^2 \text{ from } 7x^3$$
$$7x^3 - 15xy^2$$

Adding Polynomials

Combine all sets of like terms.

ex) Add $3x^2+4x-2$ and $4x^2-2x+1$

$$(3x^2+4x-2)+(4x^2-2x+1)$$

$$3x^2+4x^2+4x-2x-2+1$$

$$7x^2+2x-1$$

Can think vertically

$$\begin{array}{r} \text{ex)} \quad 3x^2+4x-2 \\ + 4x^2-2x+1 \\ \hline 7x^2+2x-1 \end{array}$$

ex) x^5+2x^3-5 and $3x^3-2x^2-3x-4$

$$\begin{array}{r} x^5 + \quad 2x^3 + \quad \quad -5 \\ + \quad \quad 3x^3 - 2x^2 - 3x - 4 \\ \hline x^5 + 5x^3 - 2x^2 - 3x - 9 \end{array}$$

$$\text{or} \\ x^5 + 2x^3 + 3x^3 - 2x^2 - 3x - 5 - 4 = x^5 + 5x^3 - 2x^2 - 3x - 9$$

Subtracting Polynomials

When subtracting polynomials, we want to subtract EVERY term!

Ⓧ Subtract $3x^2 + 2x - 5$ from $5x^2 - 2x + 7$
(want to subtract all of $3x^2 + 2x - 5$, not just the first term.)

$$\begin{array}{r} 5x^2 - 2x + 7 \\ - (3x^2 + 2x - 5) \\ \hline 2x^2 - 4x + 12 \end{array} \rightarrow \begin{array}{r} 5x^2 - 2x + 7 \\ - 3x^2 - 2x + 5 \\ \hline 2x^2 - 4x + 12 \end{array}$$

or

$$\begin{aligned} 5x^2 - 2x + 7 - (3x^2 + 2x - 5) &= 5x^2 - 2x + 7 - 3x^2 - 2x + 5 \\ &= 5x^2 - 3x^2 - 2x - 2x + 7 + 5 \\ &= 2x^2 - 4x + 12 \end{aligned}$$

Ⓧ $[(6x - 2) + (4x + 3)] - (5x + 10)$

$[6x + 4x - 2 + 3] - (5x + 10)$

$[10x + 1] - (5x + 10)$

$10x + 1 - 5x - 10$

$5x - 9$

or

$$\begin{array}{r} 6x - 2 \\ + 4x + 3 \\ \hline + - 5x - 10 \\ \hline 5x - 9 \end{array}$$