6.2
The Product Rule for Exponents

(ex) $x^3 \cdot x^4 = \underbrace{x \cdot x \cdot x}_{x^3} \cdot \underbrace{x \cdot x \cdot x \cdot x}_{x^4} = x^7$

In general

$$x^m \cdot x^n = x^{m+n}$$

(ex) $y^2 \cdot y^9 = y^{11}$

$x^{100} \cdot x^{590} = x^{690}$

$x^2 x^4 x^3 = x^9$

CAREFUL! This only works when <u>multiplying</u> with the <u>same base</u>!

So not with $x^2 \cdot y^3$ nor $x^2 + x^3$

$x \cdot x \cdot y \cdot y \cdot y$ $\quad$ $xx + xxx$

Sometimes need to rearrange factors to simplify.

(ex) $(3x^2)(2x^3) = 3 \cdot x \cdot x \cdot 2 \cdot x \cdot x \cdot x$

$= 3 \cdot 2 \cdot x \cdot x \cdot x \cdot x \cdot x$

$= 6x^5$

$(3x^2)(2x^3) = 3 \cdot 2 \cdot x^2 \cdot x^3$

$= 6x^5$

The Power Rules for Exponents

ex $(x^2)^5 = \underbrace{(x^2)(x^2)}_{x\cdot x\cdot x\cdot x}\underbrace{(x^2)}_{x\cdot x}\underbrace{(x^2)}_{x\cdot x}\underbrace{(x^2)}_{x\cdot x} = x^{10}$

In general,

$$(x^m)^n = x^{m\cdot n}$$

ex $(x^5)^6 = x^{5\cdot 6} = x^{30}$

$(x^2x^3)^6 = (x^{2+3})^6 = (x^5)^6 = x^{30}$

$(x^2)^4(x^4)^3 = (x^{2\cdot 4})(x^{4\cdot 3}) = x^8\cdot x^{12} = x^{8+12} = x^{20}$

NOTE: $(x^2)^4(x^4)^3$; $(x^2)^4$ and $(x^4)^3$ DO NOT have the same base!

ex $(2x)^4 = (2x)(2x)(2x)(2x)$

$= 2x\,2x\,2x\,2x$

$= 2\cdot 2\cdot 2\cdot 2\cdot x\cdot x\cdot x\cdot x$

$= 2^4x^4 = 16x^4$

$\left(\frac{x}{2}\right)^4 = \frac{x}{2}\cdot\frac{x}{2}\cdot\frac{x}{2}\cdot\frac{x}{2} = \frac{x\cdot x\cdot x\cdot x}{2\cdot 2\cdot 2\cdot 2} = \frac{x^4}{2^4} = \frac{x^4}{16}$

In general,

$$(xy)^n = x^n y^n \qquad \left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$$

ex) $(3x)^4 = 3^4 x^4 = 81x^4$

$(x^3y^4)^2 = (x^3)^2(y^4)^2 = x^6y^8$

$(-4x^2y^5)^3 = (-4)^3(x^2)^3(y^5)^3 = -64x^6y^{15}$

$\left(\frac{x^2}{y^3}\right)^4 = \frac{(x^2)^4}{(y^3)^4} = \frac{x^8}{y^{12}}$

$\left(\frac{2x^2x^3}{-3xy^4}\right)^2 = \frac{(2x^2x^3)^2}{(-3xy^4)^2} = \frac{(2x^5)^2}{(-3xy^4)^2} = \frac{2^2(x^5)^2}{(-3)^2(x)^2(y^4)^2} = \frac{4x^{10}}{9x^2y^8}$

The Quotient Rule

ex) $\frac{x^5}{x^2} = \frac{\cancel{x}\cdot\cancel{x}\cdot x\cdot x\cdot x}{\cancel{x}\cdot\cancel{x}} = x\cdot x\cdot x = x^3$

In general, $\frac{x^m}{x^n} = x^{m-n}$

note: need same base!

ex) $\frac{x^8}{x^3} = x^{8-3} = x^5$

$\frac{x^4x^4}{x^3} = \frac{x^8}{x^3} = x^{8-3} = x^5$

$\frac{(x^3y^2)^2}{xy^3} = \frac{(x^3)^2(y^2)^2}{xy^3} = \frac{x^6y^4}{xy^3} = x^{6-1}y^{4-3} = x^5y^1$

Summary

$x^m x^n = x^{m+n}$ $\quad (x^m)^n = x^{m \cdot n}$ $\quad (xy)^n = x^n y^n$

$\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$ $\quad \frac{x^m}{x^n} = x^{m-n}$

ex $x^3 x^4 = x^7$ $\quad (x^3)^4 = x^{12}$ $\quad (3x)^4 = 3^4 x^4 = 81x^4$

$\left(\frac{3}{x}\right)^4 = \frac{3^4}{x^4} = \frac{81}{x^4}$ $\quad \frac{x^7}{x^2} = x^5$

Multiplying Monomials

1. Multiply coefficients
2. Multiply common variables

(ex) multiply $4x^2$ and $3x^4$

$(4x^2)(3x^4) = (4 \cdot 3)(x^2 \cdot x^4) = 12 \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x$

$= 12x^6$

multiply $3x^2y^4$ and $-5xy^5$

$(3x^2y^4)(-5xy^5) = (3 \cdot -5)(x^2 x)(y^4 y^5)$

$= -15x^3y^9$

Multiplying a Polynimial and a Monomial

Use Distributive Property (Multiply over)

ex $3x^2(4x^2+3x-4) = (3x^2)(4x^2)+(3x^2)(3x)+(3x^2)(4)$
$= 12x^4+9x^3-12x^2$

$2x^2y(\frac{4}{5}xy^2 - x^2y) = (2x^2y)(\frac{4}{5}xy^2)-(2x^2y)(x^2y)$
$= \frac{8}{5}x^3y^3 - 2x^4y^2$

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Multiplying two Binomials (FOIL)

To multiply 2 Binomials use Distributive (multiply over) three times.

ex $(2x+3)(x-2) = (2x+3)(x) - (2x+3)(2)$
$= [(2x)(x)+(3)(x)] - [(2x)(2)+(3)(2)]$
$= [2x^2+3x] - [4x+6]$
$= 2x^2+3x-4x-6$
$= 2x^2-x-6$

We have a "shortcut" called FOIL!

FOIL = First multiply
+ Outside
+ Inside
+ Last

ex $(2x+3)(x-2) = \overset{\text{First}}{(2x)(x)} + \overset{\text{Outside}}{(2x)(-2)} + \overset{\text{Inside}}{(3)(x)} + \overset{\text{Last}}{(3)(-2)}$
$= 2x^2-4x+3x-6$
$= 2x^2-x-6$

ex $4(x-4)(2x-3) = 4\left[\overset{F}{x\cdot 2x} + \overset{O}{(x)(-3)} + \overset{I}{(-4)(2x)} + \overset{L}{(-4)(-3)}\right]$
$= 4[2x^2-3x-8x+12]$
$= 4[2x^2-11x+12]$
$= 8x^2-44x+48$

remember $(\_)^2$ means $(\_)(\_)$.

ex $(2x+5)^2 = (2x+5)(2x+5)$ F O I L

$= 2x\cdot 2x + 2x\cdot 5 + 5\cdot 2x + 5\cdot 5$

$= 4x^2 + 10x + 10x + 25$

$= 4x^2 + 20x + 25$

Multiply Two Polynomials

Must multiply each term in first polynomial by every term in second polynomial. Combine like terms.

ex) $(x-2)(3x^3+2x^2+x+4)$

need x times everyterm need -2 times everyterm

$x\cdot 3x^3+x\cdot 2x^2+x\cdot x+4\cdot x+-2\cdot 3x^3+-2\cdot 2x^2+-2\cdot x+-2\cdot 4$

$3x^4+2x^3+x^2+4x-6x^3-4x^2-2x-8$

$3x^4+2x^3-6x^3+x^2-4x^2+4x-2x-8$

$3x^4-4x^3-3x^2+2x-8$

ex) $(2x^2-2x-2)(x^2+3x-4)=$

$=2x^2\cdot x^2+2x^2\cdot 3x+2x^2(-4)-2x\cdot x^2-2x\cdot 3x-2x(-4)-2x^2-2\cdot 3x-2(-4)$

$=2x^4+6x^3-8x^2-2x^3-6x^2+8x-2x^2-6x+8$

$=2x^4+6x^3-2x^3-8x^2-6x^2-2x^2+8x-6x+8$

$=2x^4+4x^3-16x^2+2x+8$