Prime Factorization

Prime number - a natural number that cannot be written as the product of two other integers.

(ex) 12 is NOT prime since $12 = 3 \cdot 4$ or $12 = 2 \cdot 6$
5 is prime since no two integers multiplied together equal 5 (except $5 \cdot 1$)

Prime factorization - write a number as the product of all primes.

(ex) the prime factorization of 6 is $\boxed{2 \cdot 3}$
what about 12? $12 = 3 \cdot 4$
$= 3 \cdot 2 \cdot 2 = \boxed{2 \cdot 2 \cdot 3}$ could write $2^2 \cdot 3$
what about 210? $210 = 2 \cdot 105$
$= 2 \cdot 3 \cdot 35$
$= 2 \cdot 3 \cdot 7 \cdot 5$
$= \boxed{2 \cdot 3 \cdot 5 \cdot 7}$

The Greatest Common Factor

The biggest number that is a factor of two numbers

ex $12 = 2 \cdot 2 \cdot 3$
$210 = 2 \cdot 3 \cdot 5 \cdot 7$
They both have a 2 and a 3 so the GCF of 12 and 210 is $2 \cdot 3 = 6$.
So 6 is the largest integer that evenly divides 12 and 210 $\rightarrow \frac{12}{6} = 2 \quad \frac{210}{6} = 35$

GCF of Monomials

To find prime factorization of monomial
1. find pr. fact. of the coefficient
2. write out variables

ex $15x^2y^3 \rightarrow 3 \cdot 5 \cdot x \cdot x \cdot y \cdot y \cdot y$

To find GCF of monomials
1. find pr. fact. of each
2. List each common factor
3. find the product of these factors

ex $12x^2y^3 = 2 \cdot 2 \cdot 3 \cdot x \cdot x \cdot y \cdot y \cdot y$
$16xy^2 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot x \cdot y \cdot y$
$CF = 2 \cdot 2 \cdot x \cdot y \cdot y$

$\boxed{GCF = 4xy^2}$

Factoring Out the GCF from a Polynomial

If I have a GCF in a polynomial I can factor it out, this is distributive property in reverse.

ex) $4x(x+2) = 4x \cdot x + 4x \cdot 2 = 4x^2 + 8x$

ex) To take it back out or factor it out.

$4x^2 + 8x$

1. find GCF of $4x^2$ and $8x$

$4x^2 = 2 \cdot 2 \cdot x \cdot x$

$8x = 2 \cdot 2 \cdot 2 \cdot x$

$GCF = 2 \cdot 2 \cdot x = 4x$

2. Rewrite polynomial with GCF in it

$4x^2 + 8x = 4x \cdot x + 4x \cdot 2$

3. Take GCF out

$4x(x+2)$

Try one

$15x^2 + 6x = ?$

1. $15x^2 = 3 \cdot 5 \cdot x \cdot x$ $\quad GCF = 3 \cdot x$

$6x = 2 \cdot 3 \cdot x$

2. $3x \cdot 5x + 3x \cdot 2$

3. $3x(5x+2)$

ex. $6x^3 + 12x^2 + 3x = ?$

1. $6x^3 = 2 \cdot 3 \cdot x \cdot x \cdot x$

   $12x^2 = 2 \cdot 2 \cdot 3 \cdot x \cdot x$ $\quad GCF = 3 \cdot x$

   $3x = 3 \cdot x$

2. $3x \cdot 2x^2 + 3x \cdot 4x + 3x \cdot 1$

3. $3x(2x^2 + 4x + 1)$

Factoring Out a Negative and a Binomial

To factor out a negative, change all the signs.

ex $-x^2-4x+2$ factor out a negative.

$-1 \cdot x^2 - 1 \cdot 4x - 1 \cdot -2$

$-1(x^2+4x-2)$ or $-(x^2+4x-2)$

ex factor $-2x-2$

$(-2)\cdot x + (-2)\cdot 1$

$-2(x+1)$

To factor out a binomial, just like with a monomial

ex $(2x)x - (2x)3 = 2x(x-3)$

$(2x+1)\cdot x - (2x+1)\cdot 3 = (2x+1)(x-3)$

Factoring by Grouping

1. Group terms so that first two terms have a common factor & last two terms have common factor
2. factor out from each group
3. factor out remaining common binomial. If there is no common binomial, regroup, repeat 2 and 3.

ex $7x^2+2y+7xy+2x$

1. $7x^2+2x+7xy+2y$
2. $x\cdot 7x+x\cdot 2+y\cdot 7x+y\cdot 2$
   $x(7x+2)+y(7x+2)$
3. $(7x+2)(x+y)$

If I had done 1. $7x^2+7xy+2x+2y$
2. $7(x^2+xy)+2(x+y)$
   no common binomials, so change 1.