7.2

General Rule of Signs

$x^2 \pm bx \pm c$ { the $\pm c$ tells same or different signs
the $\pm b$ tells what same sign is

(ex) x^2+3x-4, the -4 says different $(x-)(x+)$
x^2-3x-4, the -4 says different $(x-)(x+)$
x^2+5x+4, the $+4$ says same, the $+5x$ says $+$, $(x+)(x+)$
x^2-4x+4, the $+4$ says same, the $-4x$ says $-$, $(x-)(x-)$

Idea

Reverse FOIL

(ex) $(x+2)(x+1) = x^2+1x+2x+2$
$= \underset{F}{x^2} + \underset{O+I}{3x} + \underset{L}{2}$

(ex) What if I gave you x^2+3x+2 and wanted you to find $(x+2)(x+1)$.
This is factoring trinomials.

Factoring x^2+bx+c

ex $x^2+7x+12=(x+3)(x+4)$

because $(x+3)(x+4)=x^2+4x+3x+12$

$=x^2+7x+12$

In general,

$(x+a)(x+b)=x^2+(a+b)x+ab$

So to factor a trinomial with leading coefficient 1 we need to find two numbers a & b so that $a+b$ is the middle number and $a \cdot b$ is the last number.

ex $x^2+7x+12$ $=(x+\_)(x+\_)$ I need 2 numbers that multiply to 12 and that add to 7 but

1, 12 → $1+12=13$

2, 6 → $2+6=8$

3, 4 → $3+4=7$

so use 3 and 4.

$(x+3)(x+4)$ since $(x+3)(x+4)=x^2+4x+3x+12=x^2+7x+12$

try this one

$x^2+6x+8=(x+2)(x+4)$

Factoring $x^2 - bx + c$

ex $x^2 - 5x + 6 = (x - \_\_)(x - \_\_)$

F O I L

two numbers that multiply to 6 $-1 \cdot -6 = 6$ $-2 \cdot -3 = 6$

and add to -5 $-1 + -6 = -7$ $-2 + -3 = -5$

so $(x-2)(x-3)$

check $(x-2)(x-3) = x^2 - 3x - 2x + 6 = x^2 - 5x + 6$ ✓

trinomials of the form $x^2 - bx + c$ always have $(x - \_\_)(x - \_\_)$.

try one

$x^2 - 7x + 10 = (x-2)(x-10)$

Factoring x^2+bx-c

ex $x^2+5x-6=(x-\_)(x+\_)$

two numbers multiply to -6 and add to $+5$

$-1\cdot 6$ $1\cdot -6$
$-2\cdot 3$ $2\cdot -3$

$-1+6=5$ $1+-6=-5$
$-2+3=1$ $2+-3=-1$

So $(x-1)(x+6)$

x^2+bx-c is always $(x-\ )(x+\ )$

try one

$x^2+2x-8=(x-2)(x+4)$

Factoring x^2-bx-c

ex $x^2-x-12=(x-\_\_)(x+\_\_)$

two numbers multiply to -12 and add to -1

$-1\cdot 12$ $1\cdot -12$
$-2\cdot 6$ $2\cdot -6$
$-3\cdot 4$ $3\cdot -4$

$-1+12=11$ $1-12=-11$
$-2+6=4$ $2-6=-4$
$-3+4=1$ $3-4=-1$

so $(x-4)(x+3)$

try one

$x^2-4x-12=(x-6)(x+2)$

Factoring $-x^2 \pm bx \pm c$

Need a pre-step: factor out a negative.

(ex) $-x^2 - 4x + 12$

factor out a negative

$-(x^2+4x-12) = -(x-\_\_)(x+\_\_)$

now factor $x^2+4x-12$ -12 says different

$-1 \cdot 12$ $1 \cdot -12$

$-2 \cdot 6$ $2 \cdot -6$

$-3 \cdot 4$ $3 \cdot -4$

two numbers multiply to -12

and add to $+4$ $-1+12=11$, $-2+6=4$

so $-(x-2)(x+6)$

Check: $-(x-2)(x+6) = -(x^2+6x-2x-12)$

$= -(x^2+4x-12)$

$= -x^2-4x+12$ ✓

try one

$-x^2-8x-15 = -(x+3)(x+5)$

Check: $-(x+3)(x+5) = -(x^2+5x+3x+15)$

$= -x^2-8x-15$

Prime Trinomials

Not all trinomials factor, these are called prime tri.

(ex) $x^2+5x-7 = (x+\_\_)(x+\_\_)$

two numbers multiply to -7 $\quad -1\cdot 7$ $\quad -7\cdot 1$

and add to $+5$ $\quad -1+7=+6$ $\quad -7+1=-6$

- So it's prime.

(ex) $x^2-10x-6$

- multiply $\quad -1\cdot 6$ $\quad 2\cdot -3$ $\quad 1\cdot -6$ $\quad 2\cdot -3$ $\quad$ add $\quad -1+6=5$ $\quad -2+3=1$ $\quad 1+-6=-5$ $\quad 2-3=-1$

May need to factor out the GCF first

(ex.) $3x^2+9x+6$

GCF $=3$

$3(x^2+3x+2)$

$\boxed{3(x+2)(x+1)}$

Check: $3(x+2)(x+1)=3(x^2+1x+2x+2)$

$=3(x^2+3x+2)$

$=3x^2+9x+6$ ✓

(ex.) $2x^3-4x^2-30x$

GCF $=2x$

$2x(x^2-2x-15)$

$\boxed{2x(x+3)(x-5)}$

check: $2x(x+3)(x-5)$

$2x(x^2-5x+3x-15)$

$2x(x^2-2x-15)$

$2x^3-4x^2-30x$ ✓

Factoring a trinomial with two variables.

(ex) $x^2+3xy+2y^2$

$(x+\ \ )(x+\ \ )$ I need the factors of $2y^2$

$2y^2, 1$
$y^2, 2$
$2y, y$

try them

$(x+2y^2)(x+1)=$ ~~$x^2+x+2y^2x+2y^2$~~

$(x+y^2)(x+2)=$ ~~$x^2+2x+xy^2+2y^2$~~

$(x+2y)(x+y)=$ $x^2+xy+2xy+2y^2$

$\boxed{x^2+3xy+2y^2}$

REMEMBER! ALWAYS factor out a GCF first!

(ex) $2x^3y-4x^2y^2-6xy^3$

$2xy(x^2-2xy-3y^2)$

$2xy(x+\ \ )(x-\ \ )$ factors of $3y^2$ are

$3y^2, 1$
$3y, y$
$y^2, 3$

$2xy(x+3y^2)(x-1)=$ ~~$2xy(x^2-x+3xy^2-3y^2)$~~

$2xy(x+1)(x-3y^2)=$ ~~$2xy(x^2-3xy^2+x-3y^2)$~~

$\boxed{2xy(x+y)(x-3y)}=2xy(x^2-3xy+xy-3y^2)$!