

8.2

Multiplying Rational Expressions

For a, b, c, d reals.

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$$

$$\text{(ex.) } \frac{x}{2} \cdot \frac{-3x}{8} = \frac{-3x^2}{16}$$

$$\frac{4x^2}{3y} \cdot \frac{5}{y^2} = \frac{20x^2}{3y^3}$$

$$\frac{x+2}{4x} \cdot \frac{3}{x-1} = \frac{3(x+2)}{4x(x-1)}$$

Can simplify after multiplying (sometimes).

$$\text{(ex.) } \frac{16x^2y}{2y^2z} \cdot \frac{2xyz}{8x^3z} = \frac{32x^3y^2z}{16x^3y^2z^3} = \frac{2}{z^2}$$

$$\text{(ex.) } \frac{x+2}{7x} \cdot \frac{2x-1}{4x+8} = \frac{(x+2)(2x-1)}{7x(4x+8)} = \frac{(x+2)(2x-1)}{7x \cdot 4(x+2)} = \frac{2x-1}{28x}$$

$$\text{(ex.) } \frac{x^2-5x}{x-3} \cdot \frac{x^3-9x}{x^3-5x^2} = \frac{x(x-5)x(x^2-9)}{(x-3)x^2(x-5)} = \frac{x^2(x+3)(x-3)}{x^2(x-3)} = x+3$$

Multiplying a rational expression by a Polynomial

remember, you can divide anything by 1 without changing its value, use this.

$$\begin{aligned} \text{(ex.) } \frac{x^2+3x}{x^2+8x^2} \cdot x^2+7x-8 &= \frac{x^2+3x}{x^2+8x^2} \cdot \frac{x^2+7x-8}{1} = \frac{x(x+3)(x^2+7x-8)}{x^2(x+8)} \\ &= \frac{x(x+3)(x+8)(x-1)}{x^2(x+8)} \\ &= \frac{(x+3)(x-1)}{x} \end{aligned}$$

Dividing Rational Expressions

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} \quad \text{or} \quad \frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} \cdot \frac{d}{c} \quad (\text{no } \div \text{ by } 0)$$

$$\text{(ex)} \quad \frac{16x}{13y} \div \frac{2x}{5xy} = \frac{16x}{13y} \cdot \frac{5xy}{2x} = \frac{80x^2y}{26xy} = \frac{40x}{13}$$

$$\text{(ex)} \quad \frac{\frac{x+2}{3x+5}}{\frac{3x+6}{x+1}} = \frac{x+2}{3x+5} \cdot \frac{x+1}{3x+6} = \frac{(x+2)(x+1)}{(3x+5)3(x+2)} = \frac{x+1}{3(3x+5)}$$

If you're dividing by a Polynomial do the dividing by 1 trick.

$$\begin{aligned} \text{(ex)} \quad \frac{3-x}{6-x} \div \frac{2x^2-5x-3}{1} &= \frac{3-x}{6-x} \cdot \frac{2x^2-5x-3}{1} = \frac{(3-x)(1)}{(6-x)(2x^2-5x-3)} \\ &= \frac{3-x}{(6-x)(2x+1)(x-3)} \\ &= \frac{-(x-3)}{(6-x)(2x+1)(x-3)} \\ &= \frac{-1}{(6-x)(2x+1)} \end{aligned}$$

Combined Operations

Unless there are (), do mult./div. left to right.

$$\text{(ex)} \quad \frac{2}{3x-3} \div \frac{2x+2}{x-1} \cdot \frac{5}{x+1} = \frac{2}{3x-3} \cdot \frac{x-1}{2x+2} \cdot \frac{5}{x+1} = \frac{2}{3(x-1)} \cdot \frac{x-1}{2(x+1)} \cdot \frac{5}{x+1} = \frac{1}{3(x+1)} \cdot \frac{5}{x+1} = \frac{5}{3(x+1)^2}$$