9.1

Square Roots

ex. 2 is a square root of 4 because $(2)^2=4$
3 is a square root of 9 because $(3)^2=9$
-5 is a square root of 25 because $(-5)^2=25$

In general,
b is a square root of a if $b^2=a$

ex. find two square roots of 49.
7 and -7 because $(7)^2=49$ and $(-7)^2=49$

$\sqrt{\ }$ is called a radical sign
the number inside the $\sqrt{\ }$ is the radicand.

Principal Square Root
The principal Square Root of x is the positive square root of x, denoted $\sqrt{x}$
special case: $\sqrt{0}=0$

ex. $\sqrt{49}=?$ We decided 7 and -7 were the square roots
we take the positive one for the principal sqrt.
so $\sqrt{49}=7$.

We call this simplifying the radical. ($\sqrt{49}=7$)

Sometimes we might want $-\sqrt{x}$

ex) Simplify $-\sqrt{36} = -6$

Not always integers

ex) $\sqrt{1/4} = 1/2$

not always rational

ex) $\sqrt{10} \approx 3.1622277$

To find these use calculators $\sqrt{\ }$ key.

Warning! You can't find the square root of a negative number in the reals.

ex) $\sqrt{-9}$ no # squared gives a -9.

more in ch. 10

Square Roots with Variables

For any real # x

$$\sqrt{x^2} = |x|$$

ex) Simplify:

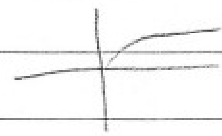
$\sqrt{36x^2} = \sqrt{(6x)^2} = |6x| = 6|x|$ $\quad$ $\sqrt{x^2+4x+4} = \sqrt{(x+2)^2} = |x+2|$

Square Root Functions

the principal square root is a function

$$f(x) = \sqrt{x}$$

The graph of $f(x)=\sqrt{x}$



Notice the domain and range of $f(x)=\sqrt{x}$
we already knew that x has to be positive (can't take $\sqrt{\ }$ of a neg. #) and that $\sqrt{\ }$, the principal root, is always positive (so y has to be positive)

Cube Roots

Just as the square root of x is a number $a : a^2 = x$

the cube root " $: a^3 = x$

Ex) 2 is a cube root of 8 since $2^3 = 8$

-3 is a cube root of -27 since $(-3)^3 = -27$

In general,

the cube root of x, $\sqrt[3]{x}$, is

$\sqrt[3]{x} = y$ if $y^3 = x$

Note that $\sqrt[3]{8} = 2$ since $2^3 = 8$

but $\sqrt[3]{8} \neq -2$ since $(-2)^3 = -2 \cdot -2 \cdot -2 = 4 \cdot -2 = -8 \neq 8$

For any real x

$\sqrt[3]{x^3} = x$

On calculator, $3\sqrt[x]{\ }$

n^{th} Roots

We talked about $\sqrt{\ }$, $\sqrt[3]{\ }$, but there are also $\sqrt[4]{\ }$ ("fourth root") $\sqrt[5]{\ }$, $\sqrt[6]{\ }$, ... or in general $\sqrt[n]{\ }$ ("n^{th} root")

If n is odd, $\sqrt[n]{\ }$ is an <u>odd root</u> and $\sqrt[n]{x^n} = x$

" "even," " <u>even root</u> and $\sqrt[n]{x^n} = |x|$

In $\sqrt[n]{x}$, n is the <u>index</u>.

ex) $\sqrt[5]{32} = 2$ since $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 32$
$\sqrt[7]{-16384} = -4$ since $(-4)^7 = -16384$
$\sqrt[6]{729} = 3$ since $3^6 = 729$

notice: $\sqrt[4]{-13}$ is not defined in reals because
no # raised to the 4^{th} is negative.

ex) $\sqrt[5]{32x^5} = \sqrt[5]{(2x)^5} = 2x$
$\sqrt[6]{729x^{12}} = \sqrt[6]{(3x^2)^6} = |3x^2| = 3x^2$