9.2

Properties of Radicals

Assume we are always in the reals. (ie no even roots of neg. no dividing by zero.)

multiplication of radicals

$\sqrt[n]{ab} = \sqrt[n]{a}\sqrt[n]{b}$

ex) $\sqrt{9\cdot 4} = \sqrt{36} = 6$

$\sqrt{9}\cdot\sqrt{4} = 3\cdot 2 = 6$

ex) $\sqrt{25x} = \sqrt{25}\sqrt{x} = 5\sqrt{x}$

ex) $\sqrt{3x}\sqrt{2y} = \sqrt{3x\cdot 2y} = \sqrt{6xy}$

note: this is NOT true for add/subt.

ex) $\sqrt{9}+\sqrt{4} \neq \sqrt{9+4}$

$3+2 \neq \sqrt{13}$

$5 \neq \sqrt{13}$

Division of radicals

$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

ex) $\sqrt{1/4} = \sqrt{1}/\sqrt{4} = 1/2$

$\sqrt{x/16} = \sqrt{x}/\sqrt{16} = \sqrt{x}/4$

$\frac{\sqrt{32x^3}}{\sqrt{2x}} = \sqrt{\frac{32x^3}{2x}} = \sqrt{16x^2} = 4x$

Simplifying Radical Expressions

A radical is simplified when

All Powers in radicand are less than the index (factor out whatever you can)

ex. Simplify. $\sqrt{20}$

$\sqrt{20} = \sqrt{4 \cdot 5} = \sqrt{4} \cdot \sqrt{5} = 2\sqrt{5}$

20 → 4, 5; 4 → 2, 2; 5 → 5, 1

$20 = 2 \cdot 2 \cdot 5 = 2^2 \cdot 5$

5 → 5, 1

Simplify $\sqrt[3]{40}$

$\sqrt[3]{40} = \sqrt[3]{2^3 \cdot 5}$

$= \sqrt[3]{2^3} \sqrt[3]{5}$

$= 2\sqrt[3]{5}$

40 → 2, 20; 20 → 2, 10; 10 → 2, 5

$40 = 2 \cdot 2 \cdot 2 \cdot 5 = 2^3 \cdot 5$

Simplify. $\sqrt{200}$

$\sqrt{2 \cdot 100} = \sqrt{2}\sqrt{100} = 10\sqrt{2}$ OR

200 → 50, 4; 50 → 25, 2; 25 → 5, 5; 4 → 2, 2

$2 \cdot 2^2 \cdot 5^2$

$\sqrt{200} = \sqrt{2 \cdot 2^2 \cdot 5^2}$

$= \sqrt{2}\sqrt{2^2}\sqrt{5^2}$

$= \sqrt{2} \cdot 2 \cdot 5 = 10\sqrt{}$

ex. $\sqrt{x^{17}} = \sqrt{x^{16} \cdot x} = \sqrt{x^{16}}\sqrt{x}$

$= \sqrt{(x^8)^2}\sqrt{x}$

$= x^8\sqrt{x}$

$\sqrt{162x^5} = \sqrt{81 \cdot 2 \cdot x^4 \cdot x} = \sqrt{81x^4 \cdot 2x} = \sqrt{81x^4}\sqrt{2x}$

$= \sqrt{(9x^2)^2}\sqrt{2x} = 9x^2\sqrt{2x}$

$x^5 = \underbrace{x \cdot x}_{x^2} \cdot \underbrace{x \cdot x}_{x^2} \cdot \underbrace{x}_{x}$

ex. $\frac{\sqrt[3]{-256x^8}}{\sqrt[3]{2x}} = \sqrt[3]{\frac{-256x^8}{2x}} = \sqrt[3]{-128x^7} = -\sqrt[3]{64 \cdot 2 \cdot (x^2)^3 x} = -\sqrt[3]{(4x^2)^3}\sqrt[3]{2x}$

$= -4x^2\sqrt[3]{2x}$

ex. $\frac{\sqrt[3]{108x^7}}{2y^9} = \sqrt[3]{\frac{54x^7}{y^9}} = \frac{\sqrt[3]{54x^7}}{\sqrt[3]{y^9}} = \frac{\sqrt[3]{2 \cdot 27 \cdot x^6 \cdot x}}{\sqrt[3]{(y^3)^3}} = \frac{\sqrt[3]{27x^6}\sqrt[3]{2x}}{y^3} = \frac{\sqrt[3]{(3x^2)^3}\sqrt[3]{2x}}{y^3}$

$= \frac{3x^2\sqrt[3]{2x}}{y^3}$