Absolute Value Equations and Inequalities

Recall from chapter 1, absolute value means the distance from a number to 0 on the number line. An example is \(|3| = 3\).

**Case 1: An absolute value equation.**
An example of an absolute value equation is \(|x| = 3\). The solutions are 3 or -3. We write the solution \(\{3, -3\}\). The solutions are 3 units to the left of 0 or 3 units to the right of 0. Graph the solution on the number line below.

![Number line graph](attachment://number_line_graph.png)

**Case 2: An absolute value inequality involving >**
An example is \(|x| > 3\). We interpret \(|x| > 3\) to mean all the numbers more than 3 units from 0 (on both sides of 0). This means \(x > 3\) or \(x < -3\). The solution to the inequality in interval notation is \((-\infty, -3) \cup (3, \infty)\). The solution is two separate intervals. Graph the solution on the number line.

![Number line graph](attachment://number_line_graph.png)
Case 3: An absolute value inequality involving <

An example is $|x| < 3$. We interpret $|x| < 3$ to mean all the numbers less than 3 units from 0 on the number line. Another way to think of this is all the numbers between -3 and 3. The solution to the inequality in interval notation is $(-3, 3)$. Graph the solution set on the number line.

![Number Line](image)

A compound inequality is another way to write the solution set: $-3 < x < 3$. This means $x > -3$ and $x < 3$. The solution is one continuous interval and was graphed above.

Remember—absolute value refers to a distance from 0 so each absolute value equation will have two parts.

Two little ways to remember: great OR less thAND

Example 1: Solve $|3x - 4| = 11$, write the solution as a set and graph it.

*Hint:* $|3x - 4| = 11$ means $3x - 4$ means $3x - 4$ $-11$ $11$

*Question:* What is the solution to the inequality $|3x - 4| < 11$? Express your answer in interval notation. Would the solution to the inequality $|3x - 4| > 11$ be different? Why or why not?
Example 2: Solve $|3x - 4| < 11$, write the solution as an inequality, an interval and graph it.

*Hint:* $|3x - 4| < 11$ means

\[
\begin{align*}
&-11 \quad \text{\textbullet} \quad 11 \\
&\text{Interval: } (-11, 11)
\end{align*}
\]
**Example 3:** Solve $|3x-4| > 11$, write the solution as an inequality, an interval and graph it.

*Hint:* $|3x-4| > 11$ means $\begin{array}{c}
3x-4 \\
\leftarrow \\
-11 \\
\end{array}$ and $\begin{array}{c}
3x-4 \\
\rightarrow \\
11 \\
\end{array}$

*Remember:* An “or” statement as in example 3 cannot be written as one continuous statement. In example 3 you wrote $3x-4 > 11$ or $3x-4 < -11$.

You cannot write $-11 > 3x-4 > 11$. This would imply $-11 > 11$. **NOT TRUE.**

You cannot write $-11 < 3x-4 < 11$. This says $3x-4 < 11$ but $3x-4 > 11$. 
Example 4: Solve $|3a + 2| + 4 = 15$

*Remember:* You must isolate the absolute value before applying the definition
Example 5: Solve $|4r - 1| = |3r + 5|$
Example 6: Solve $|−2x − 6| ≤ 5$ Write the solution in interval notation.
Example 7: Solve $|−3 + t| \geq 8$ Write the solution in interval notation.