The Slope of a Line

Slope means the steepness of something like a roof or ski slope. Think of slope as the extent to which a line slants.

**Slope** is the ratio of vertical change (rise) to horizontal change (run).

Slope is the ratio of the change in y’s to the change in x’s.

The letter \( m \) is used to denote slope.

The formula for the slope of a line through points \( x_1, y_1 \) and \( x_2, y_2 \) is

\[
m = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{y_1 - y_2}{x_1 - x_2} \quad \text{where } x_1 \neq x_2
\]

**Example 1:** Find the slope of a line through the points \( -6, 9 \) and \( 3, -3 \).
Example 2: Graph the line $3x - 4y = 12$ by graphing the intercepts. Find the slope of the line just by using the graph. Then find the slope using two points on the graph and following the slope formula.
**Example 3:** Graph $y + 3 = 0$. Using two points on the graph, find the slope of the line.

**Example 4:** Graph $x = 1$. Using the two points on the graph, find the slope of the line.
**Example 5:** Find the slope of $4x - y = 4$ using the slope formula. Use the intercepts as points.

Now solve the equation $4x - y = 4$ for $y$.

You should have $y = 4x - 4$. Notice the coefficient of $x$ is the slope of the line. Remember $y = mx + b$ from introductory algebra.
Example 6: Graph a line through the point $-3, -2$ with a slope of $\frac{1}{2}$. 
Example 7: Graph a line through the point \(1, -3\) where \(m = -\frac{3}{4}\).

Notice: a line with positive slope slants upward from left to right and a line with negative slope slants downward from left to right.

From geometry we can prove the slopes of non-vertical parallel lines are equal. We can also show the slopes of perpendicular lines are negative reciprocals of each other. In other words their product is -1.
Example 8: Determine if the lines with equations $3x + 5y = 6$ and $5x - 3y = 2$ are parallel, perpendicular or neither.

(Hint: find the slopes of each line, you can 1) use x and y intercepts for the points and use the slope formula or you can 2) rewrite your equation in $y = mx+b$ form and find the slope as the coefficient of the $x$)
Example 9: In the fall of 2006, 21.4% of ARCC students smoked. In the fall of 2008, 30.5% of ARCC students smoked. Find the average rate of change in percent per year. Let $x$ represent the year and $y$ represent the percent.